Customizing Primary Care Delivery Using E-Visits

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The demand for physician services in primary care is shaped by the number of patients associated with each physician and the frequency of scheduled office visits. While a physician can typically set the size of her patient panel without regard for individual patient preferences, office revisit intervals are determined jointly by the physician and her patients. We analyze these decisions by modeling patient demand for office visits as a function of office revisit intervals. We focus on a setting in which a physician can divert some of the patient demand away from the office visits and into the “e-visits”, which utilize less of the physician’s service capacity while still maintaining an appropriate quality of care. Using our model, we characterize care settings, defined in terms of patient panel features, parameters of primary care delivery, and physician compensation schemes, that result in increased expected earnings for the physician, larger panel size, and improved patient health. We also describe settings in which e-visits may lead to worse outcomes on at least one of these dimensions.

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1. Introduction

In the complex landscape of the US healthcare reform, one of the central issues is that of redesigning the system of delivering medical care in the presence of tens of millions of patients that, due to expanded coverage, have recently joined the healthcare system. The additional demand has spurred an active search for new approaches to care delivery that would result in better utilization of existing patient care capacity (Wishner and Burton 2017). This issue is particularly relevant for primary care, the major point of access to care for most patients.

The primary care environment is distinctively different from other, more procedural, care settings, in which an exogenously determined stream of arriving patients’ needs to be handled by one or multiple resources. By contrast, in primary care, the arrivals of patients to the practice is not just the input to operational decision making, it also is the consequence of operational decisions the practice makes. Consider the case of a diabetic patient who is presently scheduled to...
visit the practice once every four weeks (we will later on define this as the revisit interval, or RVI for short). What happens if the practice decides to see this patient every eight weeks instead? Such a change will have a direct effect on physician productivity. Holding everything else constant, the physician could now handle twice as large of a patient population. But, not everything will remain equal; our diabetic patient is now more likely to fall sick between office visits, requiring some urgent (unscheduled) visits to the practice. Such unscheduled visits are not just inconvenient (and potentially dangerous) to the patient, they also negatively impact physician capacity and the appointment planning process for the practice.

This creates a dual responsibility for a physician: overseeing healthy patients in scheduled (routine) office visits and helping unhealthy patients during unscheduled (urgent) visits. Balancing the needs of these two groups of patients has received recent attention as new technologies now enable physicians to provide some care outside of the traditional office visits. For example, virtual/telemedicine appointments between physicians and patients, also referred to as e-visits, allows a physician to provide some care without the patient coming to the practice and occupying an appointment slot. Such technologies not only alter physician productivity, but also patient utility (the patient might not like to come to the practice every four weeks) and health outcomes (unhealthy patients can be identified earlier). Moreover, depending on reimbursement policies, such technologies can also impact physician compensation. Our model explores a setting in which the physician manages patient demand by leveraging separate channels of care such as e-visits for different types of patient demand.

To analyze how the introduction of e-visits impacts the healthcare system, we need to develop a new type of model driving patient demand. This new model does not take arrivals to the practice as given, but instead explicitly models the underlying health of the patients, thereby effectively endogenizing patient demand. It also captures the different objectives of the agents involved in choosing the time of the next visit, physicians and patients.

Our approach to analyzing primary care demand treats both physicians and patients as active entities reacting to the changes in the care delivery system. In our model, patients define a range of acceptable values for their scheduled office revisit intervals (RVIs). A patient determines the range of acceptable RVI values based on the trade-off between the cost associated with making an office visit and the disutility of falling sick. On the other hand, the physician chooses the patient panel size and the RVI value to maximize her expected daily revenue subject to her daily appointment capacity. The physician’s problem is a capacity-based revenue management model (Talluri and Van Ryzin 2005), in which the physician’s revenue may be a mix of “fee-for-service” (receiving a fixed fee for each office visit), and “capitation” (receiving a fixed fee for each patient on her
panel) payments. As a result, patient demand for primary care emerges as an endogenous process governed by both patient preferences and physician financial incentives.

We focus our analysis on the channel of care customization by leveraging technological innovations such as e-visits to provide certain types of care without the need for in-office visits. Provided this technology, the physician may choose to divert to e-visits only those visits for which an e-visit and office visit offer similar quality of care. Since e-visits are likely to be less expensive for the patient than office visits, and can save the physician’s time that can be used for providing more in-office care when it is required, both parties should be willing to adopt this innovation as long as the physician receives sufficient compensation for this type of care. We demonstrate the conditions under which e-visits are feasible with respect to patient preferences and physician compensation.

Our analysis examines the impact of e-visits on three key performance measures: physician revenue, patient health, and patient panel size. These indicators reflect the impact of changes in care delivery on three major groups of stakeholders: physicians, patients, and society as a whole. We use physician revenue as an indicator of how attractive e-visits are to physicians. On the patient side, we consider a homogeneous patient panel and define patient health as the expected portion of in-office visits devoted to routine check-ups as opposed to urgent matters related to “flare-ups” in chronic conditions or acute sickness episodes. (We also study heterogeneous patient panels in Appendix B.) The changes in the panel’s overall health level reflect the impact of e-visits on patients. Finally, we treat the changes in the size of patient panel as an important societal measure of performance of the primary care system in providing primary care coverage.

We show that the endogenous nature of these patient and physician responses to e-visits has a direct impact on care outcomes. For example, the introduction of e-visits may change the degree of flexibility that patients display with respect to the range of RVI values they are willing to accept. Since in our model the RVI values are jointly selected by patients and their physicians, the balance between the scheduled and urgent visits may be altered as well, resulting in changes in patient health, as well as physician compensation, and the number of patients that a physician can accommodate on her panel. These insights reveal that ignoring the endogeneity in patient and physician responses may lead to starkly different conclusions about the impact of primary care innovations such as e-visits.

The results of our analysis can be summarized as follows:

1. We provide the analytical characterization of the physician’s optimal RVI values in the settings with and without e-visits (Propositions 1 and 4).

2. We analytically characterize the patients’ joint decision on e-visit adoption and the value of RVI (Proposition 2). We show that the “sicker” patients are more likely to adopt e-visits, and that the “healthier” patients gravitate towards e-visits only when those replace a sufficiently
large fraction of regular office visits. We also show how e-visits impact the range of RVI values that patients are willing to accept (Proposition 3).

3. We show that if e-visits, as well as routine and urgent visits, are compensated proportionally to the amount of physician capacity they consume, then their introduction increases both physician revenue and panel size with no negative impact on patient health (Proposition 5). If, however, the compensation that the physician receives for providing e-visits is not adequate, physician revenue, panel size, and patient health may all suffer (Propositions 6 and 7).

Our paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces the model at the core of the analysis. In Section 4 we model and analyze the impact of using e-visits, and in Section 5, we present the numerical results. We conclude by discussing our findings in Section 6.

2. Literature Review

In the operations management literature, several papers focus on matching patient demand and treatment capacity in primary care settings. Green et al. (2007) provide guidelines on patient panel sizing in primary care. Green and Savin (2008) and Liu and Ziya (2014) apply queuing analysis to study the effects of patient no-shows on physician panel sizes, and Zacharias and Armony (2016) study the joint problem of panel sizing and appointment scheduling. Ozen and Balasubramanian (2013) quantify the impact of case-mix on physician utilization and panel sizes, and Balasubramanian et al. (2012) show the advantages of provider flexibility and quantify the trade-off between access to and continuity of care. A key modeling element in these papers is the exogenous nature of patient demand for care. In contrast, we treat patient demand as an endogenous process governed by the physician’s choice of RVI values consistent with patient preferences.

The related literature includes a number of studies that show the impact of compensation schemes on the policies healthcare providers adopt (e.g., Adida et al. 2016, Andritsos and Tang 2018, Adida and Bravo 2018, Guo et al. 2019). Our model of physician compensation resembles the one in McGuire (2000), as it has both a fee-for-service and capitation component. Similar to Shumsky and Pinker (2003), we study the response of primary care physicians to alternative compensation schemes, but, instead of looking at the role of a physician as a gatekeeper to the healthcare system, we study the physician’s choice of office revisit intervals and patient panel size.

We also contribute to the literature that studies e-visits and telemedicine. Zhong et al. (2016) model the patient workflow in a primary care system in the presence of e-visits and evaluate the impact of a new mode of delivering care on the length of patient office visits. In our paper, we focus on the impact of e-visits on key performance indicators, such as patient health, physician revenue and patient panel size. In related work, Çakici and Mills (2016) study teletriage, a system
that provides medical advice to patients, and show that the effectiveness of teletriage depends on how patient respond to its introduction based on their perceived level of health.

Rajan et al. (2018) study the operational impact of telemedicine on a specialist serving a heterogeneous patient population suffering from chronic conditions. Our modeling approach differs from Rajan et al. (2018) in multiple ways as their model is geared toward a specialist rather than a primary care physician. First, the physician in their model optimizes patient service rate and service price, while we take these two quantities as given and focus instead on a physician’s decision on patient revisit intervals. Second, the physician compensation contract in our model is a mix of fee-for-service and capitation, a feature that is common in primary care, as compared to the fee-for-service contract considered in Rajan et al. (2018). Third, we study the impact of RVI customization as compared to uniform RVI policy that is prevalent primary care (Schectman et al. 2005). Fourth, e-visit payments by patients and e-visit compensation for the physician we consider fee-for-service and capitation elements, both of which affect the physician and patient decisions regarding RVIs, while in their model the patient is only responsible for a copay that is modeled as a fraction of the price not covered by insurance.

An emerging literature on online telemedicine platforms is topically related to our work (Liu et al. 2018, Savin et al. 2019). While this literature focuses exclusively on the online interactions between patients and providers, our work models the environment where the care delivery includes a combination of online and in-office interactions. In particular, our analysis focuses on describing the co-existence between these two modes of care delivery.

This paper is related to the work of Bavafa et al. (2019) in that both papers examine key primary care system outcomes (patient health, panel size, and physician compensation) and endogenize RVI values based on patient and physician preferences. Beyond that, the papers differ along every key dimension. For example, the present paper studies e-visits, a cost-based intervention, along with RVI customization by patient health status, whereas the focus of Bavafa et al. (2019) is on the impact of non-physician providers, a quality-based intervention. The theoretical models and subsequent results thus differ due to the separate goals of these papers. For example, the present paper examines variation in visit type compensation, physician compensation regime (allowing for combinations of fee-for-service and capitation payments), and patient heterogeneity, none of which is studied in Bavafa et al. (2019).

Our model also builds on the extant literature on preventive maintenance (McCall 1965, Wu and Zuo 2010). The problem faced by the patient is a special case of the “age replacement” policy (Glasser 1967), and the physician’s problem is related to the “machine interference” or “repairman problem” (Stecke and Aronson 1985, Cho and Parlar 1991, Haque and Armstrong 2007). In healthcare, Deo et al. (2013) study a related problem of determining revisit intervals for
asthma patients in community-based chronic care setting using a Markovian disease progression model. The novel feature of our model, not addressed in the existing literature, is the interaction between the incentives of the patient (“machine”) and the physician (“repairman”). In particular, in our model we allow not only the “repairman” but also the “machines” to actively respond to changes in care delivery, such as introduction of e-visits.

One of the key features of our model is the physician’s decision regarding the frequency of patient scheduled visits, i.e., RVI values. In particular, under RVI customization in our model, the physician’s decision on RVIs is a function of patient health. Therefore, our work is related to prior work that study the timing of disease screening and treatment. A group of studies in this literature focus on screening tests to detect the first incidence of a disease (Kirch and Klein 1974, Maillart et al. 2008, Rauner et al. 2010, Brailsford et al. 2012, Helm et al. 2015, Güneş et al. 2015, Deo et al. 2015), while another group is focused on treatment decisions for a previously detected condition (Alagoz et al. 2004, Shechter et al. 2008, Ayer et al. 2012, Lavieri et al. 2012). Our work differs from this literature since our goal is to develop insights on the interaction between patient preferences and physician incentives when deciding on patient revisit intervals rather than to develop a detailed high-fidelity model of a particular primary-care practice.

3. Traditional Mode of Care Delivery

We start the analysis by modeling the traditional mode of primary care delivery. In particular, we consider a primary care setting where a single physician provides in-office service to her patients. We focus on a homogeneous patient panel in the paper and derive results related to a heterogeneous patient panel in Appendix B. We assume that, following an office visit, a patient will fall into the sick state in the absence of care and will require an office visit after a random time period $T$. We model $T$ as taking one of the two discrete values:

$$\mathcal{T} = \begin{cases} T_l, & \text{with probability } q, \\ T_h, & \text{with probability } 1-q, \end{cases}$$

with $T_l < T_h$ and $0 \leq q \leq 1$, so that $E[T] = qT_l + (1-q)T_h$ and $\text{Var}[T] = 2(1-q)q(T_h - T_l)^2$. This simplified representation of the stochastic process of “falling sick” allows for tractable analysis of the care delivery. Also, the process described by (1) possesses the increasing failure rate (IFR) property. The IFR implies, plausibly, that, as time since the last office visit increases, the patient is more likely to get sick. When a patient falls into the sick state, he is immediately treated by the physician in an office visit during which the patient is “restored” to the healthy state, but note that our model does not rely on the patient being restored to “full health” by an office visit after falling sick. We assume that the patient is being restored to a baseline state (which we call “healthy”).
The goal of our modeling of patient health dynamics is to represent the IFR, i.e., the increased likelihood for patients to get healthier if they visit their physician more frequently. After an office visit, the patient still falls sick with probability $q$ after $T_l$ time, so, for example, having a chronic condition would still translate into a pattern of office visits. Note that although the two-point distribution in (1) is simple and allows us to make analytical progress, the three parameters $T_l$, $T_h$, and $q$ can be characterized to match the first two moments of any distribution. The two-point distribution does miss the higher moments (e.g., 3rd and 4th), but we consider this difference to be unlikely to result in qualitative changes in the insights derived from our model.

We assume that all patients on the panel transition between “healthy” and “sick” states independently from each other. The value of $q$ can be characterized as the level of overall “health” of the patients, as $T$ is stochastically decreasing in $q$, so that the lower is $q$, the healthier are the patients.

In our model, the physician chooses the RVI which is the time until the next scheduled office visit. This choice, however, depends on the health of the patient panel. The use of scheduled office visits is analogous to the “age replacement” policy in the machine maintenance literature (Glasser 1967). In primary care, patients play an active role in setting the RVI values (Welch et al. 1999), so we use a two-stage model for the process of selecting the RVI that considers both patient and physician incentives. At the first stage, patients provide the physician with a range of RVI values they find acceptable. At the second stage, the physician chooses the RVI value among the alternatives provided by the patients. The role of the patients can be thought of as that of a Stackelberg leader that provides a constraint on the RVI values for the physician’s optimization problem.

This modeling approach is similar to the classic “divide-and-choose” procedure (Brams and Taylor 1996). Note that there are other approaches to modeling the patient-physician interactions at a micro-level in this setting. One possible option is the Nash bargaining approach where the RVI value is set as a “compromise” between physician and patient preferences and is determined by optimizing a joint objective function constructed from the objectives of the two parties using a parameter that describes their relative bargaining power (Ellis and McGuire 1990). We consider the “divide-and-choose” approach to be more realistic compared to Nash bargaining in healthcare settings for two reasons. First, in practice, patients are likely to be willing to accept a range of revisit interval values instead of insisting on a single value. Second, the Nash bargaining approach relies on the knowledge of the “bargaining power” parameter that is difficult to reliably estimate in practice. Another option is a model in which the physician moves first and selects an RVI; then, the ultimate RVI value is determined after the patient adds noise to the physician-selected RVI value based on his preferences. Overall, although these models differ in their mathematical formulations, they lead to qualitatively similar outcomes.
3.1. Patient Preferences for Office Revisit Intervals

Suppose that, upon completion of every office visit of a patient, the next visit is scheduled in \( r \) time units. The patient’s actual next office visit will occur after the random time interval \( \min(T, r) \).

The expected value of this time between office visits calculated over the two-scenario distribution of \( T \) is given by

\[
T(r) = \begin{cases} 
  r, & r \leq T_l, \\
  qT_l + (1 - q)r, & T_l < r \leq T_h, \\
  qT_l + (1 - q)T_h, & r > T_h,
\end{cases}
\]

(2)

that is an increasing concave function of the RVI value \( r \). Note that, in presence of the scheduled revisit interval \( r \), every office visit falls into one of two categories we label as “routine” and “urgent”.

Under the routine visit, the patient comes to the physician’s office and is still in the “healthy” state, while under the urgent visit, the patient is in the “sick” state. For a given value of \( r \), the probability that a particular office visit falls into the “routine” category is given by

\[
\rho^r(r) = P(T \geq r) = \begin{cases} 
  1, & r \leq T_l, \\
  1 - q, & T_l < r \leq T_h, \\
  0, & r > T_h.
\end{cases}
\]

(3)

Note that for \( r = T_h \), if the patient falls sick after \( T_l \) time units and visits the physician with an urgent visit, the patient’s previous appointment corresponding to \( T_h \) is canceled. For such a patient, another appointment is scheduled for \( T_h \) time units upon the completion of the urgent visit.

We assume that during a “routine” visit to the physician’s office a patient incurs the cost \( c_o \), while during an “urgent” visit the same patient incurs the cost of \( c_o(1 + \eta) \), with \( \eta \geq 0 \). The factor \( \eta \) captures the additional cost associated with the patient being sick when visiting the office. Thus, expressed in the units of \( c_o \), the expected cost associated with an office visit is

\[
C(r) = \rho^r(r) + (1 + \eta) (1 - \rho^r(r)) = \begin{cases} 
  1, & r \leq T_l, \\
  1 + q\eta, & T_l < r \leq T_h, \\
  1 + \eta, & r > T_h.
\end{cases}
\]

(4)

Patient preferences for the RVI values are governed by the objective of minimizing the long-run average cost. We use the standard renewal process framework to calculate the patient’s long-run average cost as displayed below.

\[
D^o(r) = \frac{C(r)}{T(r)} = \begin{cases} 
  \frac{1}{r}, & r \leq T_l, \\
  \frac{1 + q\eta}{qT_l + (1 - q)r}, & T_l < r \leq T_h, \\
  \frac{1 + \eta}{qT_l + (1 - q)T_h}, & r > T_h.
\end{cases}
\]

(5)

\(^1\)Details of these calculations are in the proof of Lemma 1.
Lemma 1. For given values of $\eta$ and $q$, the global minimizer of (5) is

$$\tilde{r}^o = \begin{cases} T_l, & q > \frac{1}{1 + \frac{1}{T_l} - 1}, \\ T_h, & q \leq \frac{1}{1 + \frac{1}{T_l} - 1}. \end{cases}$$

Lemma 1 states that the patient preference for RVI values switches from the lowest possible value of $T$ to the highest possible value when his health level exceeds a certain threshold, i.e., when $q$ drops below $\frac{1}{1 + \frac{1}{T_l} - 1}$. Note that the threshold value for $q$ is a decreasing function of the “sickness factor” $\eta$, indicating that the higher is patient’s sickness cost, the easier it is for him to select more frequently scheduled office visits.

If patients knew the values of $T_l$, $T_h$, $\eta$ and $q$ with perfect precision, the outcome of the patient RVI optimization problem would result in a single RVI value. In reality, however, patients are often more “flexible” in that they are willing to accept a set of RVI values instead of just one. We model this observation about patient flexibility regarding RVI values by introducing uncertainty in the value of $\eta$. In particular, we assume that while the values of $T_l$, $T_h$, and $q$ are known to patients (as well as to their physicians), patients do not know the value of their sickness factor $\eta$ with certainty, but, rather, know that this value is located in the interval $[\eta_{\text{min}}, \eta_{\text{max}}]$. To describe this interval, we use the following notation: we denote the center of the interval with $c = \frac{\eta_{\text{min}} + \eta_{\text{max}}}{2}$, and the half-length of the interval with $c\Delta = \frac{\eta_{\text{max}} - \eta_{\text{min}}}{2}$ with $\Delta \in [0, 1]$. Using this notation, the patient knows that $\eta$ is in the interval $[c(1 - \Delta), c(1 + \Delta)]$.

Thus, in our model, the patient panel is characterized by a set of three parameters $(q, c, \Delta)$. In this set, the value of $q$ describes the degree of overall health of patient panel: high (low) values of $q$ can be characterized as “sick” (“healthy”) panels. The value of $c$ corresponds to the additional cost of a sick visit to the patient. Recall that $c$ is the notional value that represents the center of the interval for the actual patient sickness cost, $\eta$. Thus, low values of $c$ describe “stoic” patients, while high values of $c$ correspond to “worried” patients. Finally, $\Delta$ stands for the degree of flexibility that a patient displays with respect to the choice of the RVI value: low values of $\Delta$ correspond to “inflexible” patients, while high values of $\Delta$ correspond to “flexible” patients. Note that our goal here is to incorporate patient flexibility in the model rather than enabling the patients to make an optimal decision on RVI values given the distribution of $\eta$ and the patients’ risk profiles.

Based on Lemma 1, patients know that the optimal RVI value is either $T_l$ or $T_h$. Thus, given the uncertainty in the sickness factor $\eta$, patients accept the following set of RVI values depending on the flexibility parameter $\Delta$: $T_l$, $T_h$, or both. In particular, if inequality $q \leq \frac{1}{1 + \frac{1}{T_l} - 1}$ holds for any $\eta \in [c(1 - \Delta), c(1 + \Delta)]$, then the patients will select $T_h$ as their preferred RVI value. In a similar
fashion, if \( q > \frac{1}{1 + \frac{c(1-\Delta)}{T_l - 1}} \) holds for any \( \eta \in [c(1-\Delta), c(1+\Delta)] \), then the patients will select \( T_l \) as their preferred RVI value. However, if \( \frac{1}{1 + \frac{c(1+\Delta)}{T_l - 1}} \leq q \leq \frac{1}{1 + \frac{c(1-\Delta)}{T_l - 1}} \), then the patients will be willing to accept any of the two RVI values, \( T_l \) or \( T_h \). By letting \( \{r^-, r^+\} \) denote the two RVI values that patients are willing to accept, we can formally express these observations as follows.

**Lemma 2.**

\[
\{r^-, r^+\} = \begin{cases} 
\{T_h, T_h\}, & q < q^- (c, \Delta), \\
\{T_l, T_l\}, & q > q^+ (c, \Delta), \\
\{T_l, T_h\}, & q^- (c, \Delta) \leq q \leq q^+ (c, \Delta),
\end{cases}
\]  

(7)

where

\[
q^- (c, \Delta) = \frac{1}{1 + \frac{c(1+\Delta)}{T_l - 1}},
\]

(8)

\[
q^+ (c, \Delta) = \frac{1}{1 + \frac{c(1-\Delta)}{T_l - 1}}.
\]

(9)

Lemma 2 describes the impact of the level of health \( q \) on the RVI values acceptable for a particular patient panel. In particular, if patient health level is low (high), the patient panel is “inflexible” and will insist on the lowest (highest) RVI value. On the other hand, for the health levels in the intermediate range, patients are “flexible” with respect to the choice of the RVI values, allowing the physician to make that choice according to her preferences.

### 3.2. Appointment Capacity Allocation and Physician Compensation Schemes

We consider a physician who serves a homogeneous panel of \( N \) patients. Note that for simplicity we assume that \( N \) can take fractional values. We also assume that demand for the physician’s services is sufficiently high, and the physician is able to select the overall size of her patient panel \( N \). The physician has to provide sufficient daily appointment capacity to deal with the total expected daily demand from all patient. Using the standard renewal process framework (similar to (2)-(5)), the physician’s capacity constraint is given by

\[
N \left( \rho^r (r) \tau^r + (1 - \rho^r (r)) \tau^u \right) \leq A,
\]

(10)

where \( \tau^r (\tau^u) \) is the time required by a routine (urgent) patient visit, and \( A \) is the physician’s total daily service capacity. We assume that an urgent visit requires longer time commitment from a physician, so that \( \tau^u > \tau^r \).

In choosing the size of her patient panel \( N \) and the revisit intervals \( r \), a physician is guided by her compensation scheme as has been shown by prior work, including Hickson et al. (1987), Gosden et al. (2001), Lee et al. (2010), Shumsky and Pinker (2003). Similar models have been developed.
in the operations management literature focusing on fee-for-service and capitation contracts in healthcare (Andritsos and Tang 2018, Adida and Bravo 2018). Note that while models that consider a revenue-maximizing physician are common in the literature (e.g., Gupta and Wang 2008, Liu 2016), other terms such as patient health or social welfare may appear in the objective function of the physician.

In our analysis, we focus on two common incentive schemes: fee-for-service (FFS) and capitation (CAP). We assume that under the fee-for-service incentive scheme a physician is paid a fixed amount $R_r$ for each routine visit and $R_u$ for each urgent visit. An urgent visit, requiring more effort and a longer time commitment from a physician, is compensated at a higher rate, i.e., $R_u > R_r$. For example, Medicare and Medicaid FFS payments increase as a function of visit complexity: “established patient” visits can be billed under CPT\(^2\) codes 99211-99215 (Brunt 2011).

The expected daily compensation for a fee-for-service physician is (using the renewal process framework)

$$
\Pi_{FFS}(N, r) = N \left( \rho^r(r) \frac{R_r}{T(r)} + \left(1 - \rho^r(r)\right) \frac{R_u}{T(r)} \right),
$$

(11)

Under the capitation scheme a physician is paid a fixed amount (per time period, e.g., a year) for each patient on her panel. Thus, a “capitation” physician, effectively, focuses on maximizing the size of her patient panel $N$:

$$
\Pi_{CAP}(N, r) = NR^d,
$$

(12)

where $R^d$ is the fixed daily compensation for each patient.

The physician’s total compensation may be a combination of fee-for-service and capitation components that reflects a mix of insurance policies used by the patients on her panel:

$$
\Pi_\delta(N, r) = \delta \Pi_{FFS}(N, r) + (1 - \delta) \Pi_{CAP}(N, r)
= N \left( (1 - \delta) R^d + \delta \left( \frac{\rho^r(r) R_r}{T(r)} + \left(1 - \rho^r(r)\right) \frac{R_u}{T(r)} \right) \right),
$$

(13)

where $\delta$ refers to the proportion of physician daily compensation that is based on the FFS scheme. For example, if $\delta = 0$, the physician’s compensation scheme is a pure capitation, and if $\delta = 1$, the physician’s compensation scheme is a pure fee-for-service. Note that all the revenue items in the model (i.e., $R_r$, $R_u$, and $R^d$) are paid by the patient’s insurance company as opposed to by the patient himself. Patient’s out-of-pocket expenses can be modeled as part of the patient cost shown in (4). Also, while the capitation payments are not dependent on patient health or whether the visits are routine or urgent, the fee-for-service payments are weighed by the fraction of patient visits that are routine because routine and urgent visits are compensated at different rates.

In summary, the problem of selecting patient panel size \( N \) and the office revisit interval \( r \) that a physician faces can be formulated as

\[
\max_{N,r} \Pi_\delta \left( N, r \right) \tag{14}
\]

subject to

\[
N \left( \rho^r \left( r \right) \tau^r + \left( 1 - \rho^r \left( r \right) \right) \tau^u \right) \leq A, \tag{15}
\]

\[
r \in \left\{ r^-, r^+ \right\}. \tag{16}
\]

In this model, (14)-(16) reflect the “divide-and-choose” approach to selecting the RVI values, where a physician chooses the optimal values among the ones acceptable to patients. We use the notation \( \left\{ \hat{N}, \hat{r} \right\} \) to denote the values of patient panel size and revisit intervals that optimize (14)-(16). To simplify the analysis, we will assume that the daily service capacity \( A \) and the patient panel size \( N \) can take fractional values. Below we describe the values of \( \left\{ \hat{N}, \hat{r} \right\} \) for the setting with “flexible” patient. To provide the analytical characterization of the optimal solution to (14)-(16), we introduce the following quantities:

\[
\bar{q}^\tau = \frac{1}{1 + \left( \frac{\tau^u - 1}{\tau^T - 1} \right)^{\delta R}}, \tag{17}
\]

\[
\bar{q}^R = \frac{1}{1 + \left( \frac{\tau^u - 1}{\tau^T - 1} \right)^{\delta R}}, \tag{18}
\]

\[
Q^T = \frac{T_h}{T_l} - 1, \tag{19}
\]

\[
\bar{R} = 1 + \frac{(1 - \delta) R^d T_l}{\delta R^r}, \tag{20}
\]

\[
\Sigma = \left( 1 - \frac{\bar{q}^\tau}{\bar{q}^R} \right) \left( \frac{\delta R^r}{(1 - \delta) R^d T_l} \right). \tag{21}
\]

The quantities in (17)-(21) appear in future analytical derivations. The value of \( \bar{q}^\tau \) is a measure of heterogeneity in terms of the time that the physician has to invest on routine and urgent visits. In a similar fashion, \( \bar{q}^R \) measures heterogeneity in the revenue generated by routine and urgent visits. \( Q^T \) measures the spread of the RVI values for routine and urgent visits, e.g., if \( T_h \) and \( T_l \) values are close, \( Q^T \to +\infty \), while if they are far apart, \( Q^T \to 1 \). Both \( \bar{R} \) and \( \Sigma \) are composite measures that describe the contributions of the capitation and fee-for-service elements of physician compensation. \( \bar{R} \) measures solely the revenue aspects and is monotone in \( \delta \), e.g., as \( \delta \to 0 \) we have \( \bar{R} \to +\infty \), and as \( \delta \to 1 \) we have \( \bar{R} \to 1 \). The expression for \( \Sigma \), however, includes \( \bar{q}^\tau \), \( \bar{q}^R \), and \( \bar{R} \); therefore, it is a composite measure that combines \( \bar{R} \) with the factors that describe heterogeneity in revenue and capacity consumption between urgent and routine visits.
Proposition 1. Consider a patient panel with \( q \in [q^-(c, \Delta), q^+(c, \Delta)] \). Then, the optimal RVI values in (14)-(16) are given by

\[
\hat{r} = \begin{cases} 
T_h, & q \leq \frac{q^r}{(1+\Sigma)^+}, \\
T_l, & \text{otherwise},
\end{cases}
\]

where \( x^+ = \max(x, 0) \).

For a flexible patient panel, Proposition 1 describes the optimal RVI value chosen by the physician. In particular, the physician uses a threshold strategy: if the panel is healthier than a threshold (i.e., \( q \leq \frac{q^r}{(1+\Sigma)^+} \)), the physician picks the large RVI \( (T_l) \); otherwise, she picks the small RVI \( (T_h) \). In (22), the term \((1 + \Sigma)^+\) is driven by the relative contributions of the capitation and fee-for-service elements of physician compensation, e.g., if \( \delta \to 0 \), then \( \Sigma \to 0 \) and \((1 + \Sigma)^+ \to 1 \). The proposition also specifies how the panel health threshold depends on the key parameters in the physician’s problem: \( \tau^r, \tau^u, R^r, R^u, T_h, T_l \), and \( \delta \). For example, holding all other parameters constant, the panel health threshold is a decreasing function of \( \frac{\tau^r}{\tau^u} \), i.e., as urgent visits become longer compared to routine visits, the physician becomes more inclined to choosing the smaller RVI value.

Another insight from Proposition 1 is that the physician may choose \( T_l \) to increase panel size. Consider the case of a physician that is entirely compensated on a capitation basis (i.e., \( \delta = 0 \)) and hence is incentivized to increase panel size. In this case, \( \Sigma = 0 \) in (22), so the physician picks \( T_l \) if the patient panel is sufficiently sick, i.e., \( q > \bar{q}^+ \). To see the intuition behind this result, suppose the physician picks an RVI of \( T_h \) for an excessively sick patient population. Most patients will fall sick before their routine appointment and need an urgent appointment; because urgent appointments consume more of the physician’s capacity compared to the routine appointments (i.e., \( \tau^u > \tau^r \)), the physician capacity for handling patients is reduced.

As we show below, the expressions for the optimal RVI decisions simplify in the settings with “proportional” compensation rates, where \( \frac{R^r}{\tau^r} = \frac{R^u}{\tau^u} \).

Corollary 1. Consider a heterogeneous patient panel with \( q \in [q^-(c, \Delta), q^+(c, \Delta)] \) under the “proportional” compensation \( (\frac{R^r}{\tau^r} = \frac{R^u}{\tau^u}) \). Then, the optimal RVI values in (14)-(16) are given by

\[
\hat{r} = \begin{cases} 
T_h, & q \leq \bar{q}^+, \\
T_h, & \text{otherwise}.
\end{cases}
\]

As Corollary 1 states, under the “proportional” compensation for urgent and routine visits, the physician’s choice of RVI value is only a function of \( q, T_l, T_h, \tau^r, \) and \( \tau^u \). In particular, whether the physician decides to see the patients as often or as infrequently as possible is governed exclusively by their health level, with the value of \( \bar{q}^+ \) serving as a “switching threshold”. Note that under proportional compensation, \( \bar{q}^+ = \bar{q}^R \) and hence \( \Sigma = 0 \). The “proportional” compensation makes the fee-for-service side of the physician incentives indifferent toward RVI values, so the physician focuses on the RVI value that maximizes the panel size.
4. Customizing Care Using E-Visits

In this section, we will look at how the introduction of e-visits alters the choice of revisit intervals for different patient groups, the total size of patient panel, as well as physician compensation. On the physician side, e-visits are characterized by the service time $\tau^e < \tau^r$ they require, and the per-visit, fee-for-service compensation $R^e < R^r$, as well as the daily capitation compensation $R^d < R^d$.

In the presence of e-visits, a fraction of patient demand for primary care is safely handled without patients having to come to the physician’s office. We assume that the quality of online visits for this group of patient care requests is the same as the quality of face-to-face office visits. There is evidence that such tasks exist in primary care (Pelak et al. 2015), e.g., medication review. In our model, we also assume that only a fraction of routine office visits fit into such e-visit category and that all urgent visits must still be handled at the office. While we believe that the latter is a realistic assumption, our model can be readily extended to the case where a finite fraction of urgent visits can also be attended to remotely. In the model, the patient and physician attempt to conduct a fraction of routine visits, $\alpha^e$, via e-visits. Of these attempts, a fraction $\beta^e$ are successful (i.e., the visit can be completely handled by e-visits), and a fraction $1 - \beta^e$ require an office visit in addition to the e-visit. Therefore, there are three possibilities for routine visits: (a) a fraction $\alpha^e \beta^e$ are replaced by e-visits, (b) a fraction $\alpha^e (1 - \beta^e)$ require an e-visit and an office visit, and (c) the remainder are handled entirely via office visits.

We consider a general setting where “diverting” some of the routine visits to an “e-visit” channel of care, on the one hand, saves a physician some service time that can be allocated to more demanding and urgent cases, while, on the other hand, leading to a potential revenue loss due to lower compensation that “e-visits” may bring. At present, insurance companies and Centers for Medicare & Medicaid Services (CMS) are experimenting with different forms of payment for e-visits. A number of physician practices are experimenting with charging patients fixed annual fees (an analogue of $R^d$) as well as per-e-mail fees (an analogue of $R^e$) in exchange for offering e-visits (Reijonsaari et al. 2005, Fairview Health Services 2013). Similarly, CMS has been offering compensation plans such as Chronic Care Management (CCM) that charge the patients who sign-up a monthly fee (Twiddy 2015).

We treat $\alpha^e$ as an “average” (over a patient panel) fraction of routine visits that are attempted to be replaced by “e-visits”. Similarly, $\beta^e$ represents the average fraction of such attempts that are successful. In the absence of insurance support, the entire physician compensation for providing e-visits comes from patient out-of-pocket payments, and, as we discuss later, patients can choose not to adopt e-visits. Note that, even in the absence of out-of-pocket costs, a patient may not be able to use e-visit as a substitute for all routine visits, since some routine visits may require
in-office care. Thus, we assume that $\alpha^r_e < 1$. Further, if a patient avoids coming to the office for the routine visit and uses an e-visit instead, he does not incur the cost of visit $c_o$. Note that the cost of coming to the physician’s office is more that the cost of an e-visit (i.e., $R^e < c_o$).

E-visits alter the trade-off that a physician faces in allocating her service capacity. In particular, the physician’s objective function changes from (13) to

$$\Pi^e_\delta(N, r) = \delta \Pi^e_{FFS}(N, r) + (1 - \delta) \Pi^e_{CAP}(N, r)$$

$$= N \left( (1 - \delta) \bar{R}^e + \delta \left( \frac{\rho^r(r) \bar{R}^e + (1 - \rho^r(r)) R^e}{T(r)} \right) \right),$$

where

$$\bar{R}^e = \alpha^r_e \beta^r_e R^e + \alpha^r_e (1 - \beta^r_e) (R^e + R^r) + (1 - \alpha^r_e) R^r = \alpha^r_e R^e + (1 - \alpha^r_e \beta^r_e) R^r,$$

$$\bar{R}^d = R^d,$$

(24)

The physician also faces the following capacity constraint

$$N \left( \frac{\rho^r(r) \bar{\tau}^e + (1 - \rho^r(r)) \bar{\tau}^u}{T(r)} \right) \leq A,$$

(27)

where

$$\bar{\tau}^e = \alpha^r_e \beta^r_e \tau^e + \alpha^r_e (1 - \beta^r_e) (\tau^e + \tau^r) + (1 - \alpha^r_e) \tau^r = \alpha^r_e \tau^e + (1 - \alpha^r_e \beta^r_e) \tau^r.$$

(28)

We will denote the RVI values optimizing (38)-(40) as $\hat{\tau}^e$. Note that for $\alpha^r_e \in (0, 1)$, $\bar{R}^e < R^e$ iff $R^e < \beta^r_e R^r$, and $\bar{\tau}^e < \tau^e$ iff $\hat{\tau}^e < \beta^r_e \tau^e$. These two conditions can be combined into:

$$\beta^r_e > \max \left\{ \frac{R^e}{R^r}, \frac{\bar{\tau}^e}{\tau^e} \right\}.$$  

(29)

The condition in (29) is likely to hold in most practical settings. For example, consider a case in which $\beta^r_e = 0.5$, i.e., only 50% of e-visits successfully avoid an office visit; the other half lead to an office visit in addition to the e-visit. In such a setting, (29) is not satisfied only if either of the following is true: (i) e-visit compensation is more than half of an office visit compensation, or (ii) e-visits take longer than half of an office visit. In other words, if most e-visit attempts fail ($\beta^r$ is small) but e-visits are generously compensated or take a lot of time from the physician, the condition in (29) breaks down. Additionally, from a patient’s standpoint, if $\beta^r$ is small but $R^e$ is large, patients will not adopt e-visits either because they become too costly (we discuss this in Proposition 2).

While successful e-visits allow patients to avoid the cost $c_o$ of coming to the office for some routine visits, patients still incur the cost $c_o(1 + \eta)$ for an unscheduled visit. Thus, if the scheduled
office revisit interval for patients is set at \( r \), the expected visit cost, in units of \( c_o \), in the presence of e-visits is

\[
C^e(r) = \frac{R_e}{c_o} \alpha_e \rho^e(r) + (1 - \alpha_e \beta_{re}) \rho^e(r) + (1 + \eta)(1 - \rho^e(r))
\]

\[
= \begin{cases} 
1 - \left( \beta_{re} - \frac{R_e}{c_o} \right) \alpha_e, & r \leq T_l, \\
1 - \left( \beta_{re} - \frac{R_e}{c_o} \right) \alpha_e, & T_l < r \leq T_h, \\
1 + \eta, & r > T_h.
\end{cases}
\]  

(30)

Similar to (5), we can define the expected daily cost for a patient as

\[
D^e(r) = \frac{R_d}{c_o} + \frac{C^e(r)}{T(r)} = \begin{cases} 
\frac{R_d}{c_o} + \frac{1 - \left( \beta_{re} - \frac{R_e}{c_o} \right) \alpha_e}{1 - (1 - q) \left( \beta_{re} - \frac{R_e}{c_o} \right) \alpha_e + q(1 + \eta)}, & r \leq T_l, \\
\frac{R_d}{c_o} + \frac{1 + \eta}{T_l + (1 - q) T_h}, & T_l < r \leq T_h, \\
\frac{R_d}{c_o} + \frac{1 + \eta}{T_l + (1 - q) T_h}, & r > T_h.
\end{cases}
\]  

(31)

The RVI value that minimizes \( D^e(r) \) is denoted by \( \bar{r}^e \).

### 4.1. Patient and Physician Responses to E-Visits

The patient has the option to not sign up for e-visits. In such a case, the patient’s expected daily cost is given by (5). We model this by

\[
\bar{\theta} = \begin{cases} 
1, & D^e(\bar{r}^e) \leq D^o(\bar{r}^o), \\
0, & \text{o.w.},
\end{cases}
\]  

(32)

where the patient’s long-run average cost is given by

\[
D(\bar{\theta}, r) = \begin{cases} 
D^e(r), & \bar{\theta} = 1, \\
D^o(r), & \bar{\theta} = 0.
\end{cases}
\]  

(33)

The patient’s preference regarding RVI value and e-visit adoption are given by the following result, where \( \bar{\theta} \) and \( \bar{r}^e \) represent the patient’s decisions regarding e-visit adoption and RVI value, respectively.

**Proposition 2.** Under e-visits, for given values of \( \alpha_e, \beta_{re}, \eta, \) and \( q \), the global minimizers of the patient’s long-run average cost, \( D(\bar{\theta}, r) \), are
capitation fee, patients will adopt e-visits (for any routine visits, so the e-visit cost savings are more substantial to them. Note that without the e-visit the impact factor of e-visits is high). In particular, sicker patients tend to need more frequent toward e-visits when a relatively large fraction of office visits are replaced by e-visits (i.e., when relatively sicker patients are more likely to adopt e-visits, and healthier patients only gravitate such conditions. Also, when patients opt to adopt e-visits, as e-visit impact factor increases patients when the patient knows the value of \( \eta \)

The patient has four possible choices which stem from the combination of two decisions: e-visit adoption (i.e., \( \bar{\theta} \in \{0, 1\} \)) and RVI value (i.e., \( \bar{r} = \{T_l, T_h\} \)). Proposition 2 describes patient’s choice when the patient knows the value of \( \eta \) with certainty. For example, the first case in (34) describes the case in which the patient is sufficiently sick and the e-visit impact factor, \( \left( \beta_e^r - \frac{R^c_e}{c_0} \right) \alpha_e^r \), is sufficiently low. Under such conditions, the patient chooses to not adopt e-visits and picks an RVI value of \( T_l \). We further illustrate the results of Proposition 2 using Figure 1.

Figure 1a illustrates the conditions described in (34). In particular, for sufficiently low values of e-visit impact factor, \( \left( \beta_e^r - \frac{R^c_e}{c_0} \right) \alpha_e^r \), patients choose not to adopt e-visits: if e-visits are not sufficiently effective in replacing office visits, e-visit adoption increases patient cost in the presence of the e-visit capitation fee \( (R^d_e) \). The areas labeled with \( (\bar{\theta}, \bar{r}) = (0, T_l) \) and \( (\bar{\theta}, \bar{r}) = (0, T_l) \) represent such conditions. Also, when patients opt to adopt e-visits, as e-visit impact factor increases patients gravitate toward more frequent scheduled routine visits (in-office and e-visit). Figure 1a also shows that relatively sicker patients are more likely to adopt e-visits, and healthier patients only gravitate toward e-visits when a relatively large fraction of office visits are replaced by e-visits (i.e., when the impact factor of e-visits is high). In particular, sicker patients tend to need more frequent routine visits, so the e-visit cost savings are more substantial to them. Note that without the e-visit capitation fee, patients will adopt e-visits (for any \( q \)) as they are inherently cost saving \( (R^c_e < c_0) \).

As before, patients know that their sickness factor, \( \eta \), is in the interval \([c(1 - \Delta), c(1 + \Delta)]\). Therefore, they may be flexible in terms of the RVI values that they find acceptable. When e-visits

\[
\begin{align*}
(\bar{\theta}, \bar{r}) &= \begin{cases}
(0, T_l), & q > \frac{1}{1 + \frac{\eta \beta_e^r}{\beta_e^l}} - \frac{R^d_T}{c_0}, \\
(0, T_h), & q \leq \frac{1}{1 + \frac{\eta \beta_e^r}{\beta_e^l}} - \frac{R^d_T}{c_0} \left( q T_l + (1 - q) T_l \right),
\end{cases} \\
&\cup \left\{ \begin{aligned}
q &< \frac{1}{1 + \frac{\eta \beta_e^r}{\beta_e^l}} - \frac{R^d_T}{c_0} - \frac{1}{1 + \frac{\eta \beta_e^r}{\beta_e^l}} \left( q T_l + (1 - q) T_l \right) \\
&\left( \beta_e^r - \frac{R^c_e}{c_0} \right) \alpha_e^r < 1 + T_l \left( \frac{R^d_T}{c_0} - \frac{1 + \eta \beta_e^r}{q T_l + (1 - q) T_l} \right)
\end{aligned} \right\}.
\end{align*}
\]

\[(34)\]

where

\[
\eta_e^r = \frac{\eta + \left( \beta_e^r - \frac{R^c_e}{c_0} \right) \alpha_e^r}{1 - \left( \beta_e^r - \frac{R^c_e}{c_0} \right) \alpha_e^r}.
\]

\[(35)\]
are introduced, patients decide on e-visit adoption and the acceptable range of RVIs simultaneously. We model this in the following way: patients adopt e-visits only if they decrease the patients’ long-run average costs for all values of $\eta \in [c(1-\Delta), c(1+\Delta)]$. Figure 1b shows how patients’ choice of e-visit adoption and their acceptable range of RVI values change as they become flexible ($\Delta = 0.9$). In the area marked as $(1,T_h) \sim (1,T_l)$ patient choose to adopt e-visits and accept both RVI values of $T_l$ and $T_h$, and in the area marked as $(0,T_l) \sim (0,T_h)$ patients do not adopt e-visits but are flexible with both RVI values of $T_l$ and $T_h$. In the rest of the areas, patients are inflexible with respect to RVI but may choose to adopt e-visits or not.

The areas where patients are flexible in Figure 1b become narrower as the e-visit impact factor increases. This is because e-visits lead to RVI inflexibility with $T_l$ for the relatively sicker patients as the cost of a routine e-visits is smaller than the cost of a routine office visits. We also observe that in Figure 1b the area where patients adopt e-visits and remain flexible with respect to RVI values is in the middle of the plot: if e-visits are not effective enough in terms of replacing office visits, patients do not adopt e-visits, and if e-visits are too effective in replacing office visits, patient adopt e-visits but become inflexible with the RVI of $T_l$. Note that Figure 1 is drawn for $\beta_e^r = 1$. If $\beta_e^r < 1$ the y-axis in the figure shrinks with its maximum at $\beta_e^r$. This is intuitive as decreases in $\beta_e^r$ translate into lower e-visit effectiveness in terms of substituting office visit.

Similar to (8)-(9), if patients adopt e-visits there exist “critical” threshold values for patient health level that separate “flexible” patients from “inflexible” ones:

$$q_e^-(c, \Delta, R_e^r, \alpha_e^r, \beta_e^r) = \frac{1}{1 + \frac{(\beta_e^r - \frac{R_e^r}{c_e^r}) \alpha_e^r}{(1 - (\beta_e^r - \frac{R_e^r}{c_e^r}) \alpha_e^r) (\frac{T_h}{T_l} - 1)}}$$

(36)

Figure 1: Patient’s choice regarding e-visit adoption, $\bar{\theta}$, and acceptable range of RVI, $r$, as a function of patient health levels, $q$, and e-visits impact factor, $\left(\beta_e^r - \frac{R_e^r}{c_e^r}\right)\alpha_e^r$, for two values of patient flexibility, $\Delta$

(a) $\Delta = 0$

(b) $\Delta = 0.9$
\begin{equation}
q_c^+ (c, \Delta, R_e, \alpha_e, \beta_e) = \frac{1}{1 + \frac{c(1-\Delta) + (\beta_e - \frac{R_e}{\alpha_e})}{\alpha_e(\frac{T_h}{T_l} - 1)}}.
\end{equation}

We now describe the physician’s problem under e-visits. The problem can be characterized as

\begin{equation}
\max_{N,r} \Pi_e^\delta(N,r) \tag{38}
\end{equation}

\begin{equation}
s.t. \quad N \left( \left( \frac{1}{q} - \left( \frac{T_h}{T_l} - 1 \right) \right) - c(1-\Delta) \right), \tag{39}
\end{equation}

\begin{equation}
r \in \{ T_l, T_h \}, \tag{40}
\end{equation}

where the optimization includes e-visits only if patients choose to adopt it as described in Proposition 2.

As we discuss below, the introduction of e-visits can transform a flexible patient group into an inflexible one, and vice versa. For this analysis, it is convenient to introduce the following threshold values for \( \alpha_e \):

\begin{equation}
\bar{\alpha}_e^r (q) = \left( \left( \frac{1}{q} - \left( \frac{T_h}{T_l} - 1 \right) \right) - c(1-\Delta) \right) \left( \frac{\beta_e - \frac{R_e}{\alpha_e}}{\alpha_e} \right), \tag{41}
\end{equation}

\begin{equation}
\alpha_e^r (q) = \left( \left( \frac{1}{q} - \left( \frac{T_h}{T_l} - 1 \right) \right) - c(1+\Delta) \right) \left( \frac{\beta_e - \frac{R_e}{\alpha_e}}{\alpha_e} \right). \tag{42}
\end{equation}

**Proposition 3 (patient flexibility with e-visits).** Consider a setting where patients choose to adopt e-visits. a) A flexible patient panel remains flexible upon the introduction of e-visits if and only if \( q^- (c, \Delta) \leq q \leq q^+ (c, \Delta) \) and \( \alpha_e \leq \bar{\alpha}_e^r (q) \).

b) A flexible patient panel becomes inflexible upon the introduction of e-visits if and only if \( q^- (c, \Delta) \leq q \leq q^+ (c, \Delta) \) and \( \alpha_e^r > \bar{\alpha}_e^r (q) \).

c) An inflexible patient panel becomes flexible upon the introduction of e-visits if and only if \( q < q^- (c, \Delta) \) and \( \alpha_e^r (q) \leq \alpha_e \leq \bar{\alpha}_e^r (q) \).

Part a) of Proposition 3 highlights the case in which patient flexibility regarding RVI values is maintained following the introduction of e-visits. Parts b) and c), however, describe the critical levels of the e-visit “impact factor” \( \left( \frac{\beta_e}{\alpha_e} - \frac{R_e}{\alpha_e} \right) \alpha_e^r \) that change the flexibility of the patient panel with respect to the RVI values.

Figure 2 illustrates the results of Proposition 3 for a patient group with the expected sickness factor \( c = 2 \), flexibility parameter \( \Delta = 0.5 \) and the shortest and the longest “sickness” times of \( T_l = 90 \) and \( T_h = 360 \), respectively. In this figure, the e-visit impact factor \( \left( \frac{\beta_e}{\alpha_e} - \frac{R_e}{\alpha_e} \right) \alpha_e^r \) is allowed...
Figure 2: RVI flexibility for different patient health levels $q$ as a function of e-visits impact factor $\left(\beta_e - \frac{R_e}{c_0}\right)\alpha_e$

$\left(c = 2, \Delta = 0.5, \beta_e = 1, T_h = 360, T_l = 90\right)$.

to vary between 0 (“unattractive” e-visits) to 1 (costless e-visits that perfectly replace all routine visits). The results of part a) and part b) of Proposition 3 are illustrated by the behavior of patients with the “intermediate” level of health, $q = 0.6$. These patients remain flexible while the attractiveness of e-visits and their impact is low. They become inflexible and opt for most frequent scheduled visits as the “e-visit” channel becomes more attractive. On the other hand, patients with “low” health level ($q = 0.4$) exhibit behavior described in part c) of Proposition 3: while being inflexible and insisting on the most infrequent scheduled visits in the absence of e-visit channel, these patients become flexible once the level of attractiveness of e-visits rises. As the e-visits impact grows further, such patients may become inflexible again, but this time opting for the most frequent scheduled visits.

Similar to (17)-(20), we will use the following quantities to characterize the optimal solution to (38)-(40):

$\bar{q}_e^- = \frac{1}{1 + \left(\frac{\beta_e - R_e}{\alpha_e^e} - 1\right)}$, \hspace{1cm} (43)

$\bar{q}_e^R = \frac{1}{1 + \left(\frac{\beta_e - R_e}{\alpha_e^e} - 1\right)}$, \hspace{1cm} (44)

$\bar{R}_e = 1 + \frac{(1 - \delta) \bar{R}_e^e T_i}{\delta \bar{R}_e^e T_l}$, \hspace{1cm} (45)

$\Sigma_e = \left(1 - \frac{\bar{q}_e^-}{\bar{q}_e^R}\right) \left(\frac{\delta \bar{R}_e^e T_l}{(1 - \delta) \bar{R}_e^e T_l}\right)$, \hspace{1cm} (46)
These quantities are the e-visit parallels of the ones we described earlier in (17)-(21).

**Proposition 4.** Consider a setting where patients choose to adopt e-visits, the patient panel is homogeneous with 
\[ q \in [q^-(c, \Delta), q^+(c, \Delta)], \]
and (29) holds.

a) Suppose that 
\[ \alpha^e_c \leq \bar{\alpha}^e_c(q) . \]
Then, the optimal RVI values in (38)-(40) are given by
\[
\hat{r}^e = \begin{cases} 
T_h, & q \leq \frac{q^0}{(1+\Sigma_e)^+} , \\
T_l, & \text{otherwise},
\end{cases}
\] (47)
where \( x^+ = \max(x, 0) \).

b) Suppose that 
\[ \alpha^e_c > \bar{\alpha}^e_c(q) . \]
Then the optimal RVI values in (38)-(40) are given by 
\[ \hat{r}^e = T_l . \]

The physician’s choice of RVI under e-visits are presented in Proposition 4. In case a) the patient stays flexible after e-visits, and the physician has a choice in the RVI values, while in case b) the patient is inflexible with an RVI of \( T_l \). We divide the paper’s results into two sets. The first set of results relate to case a): Propositions 5 (proportional e-visit compensation) and 6 (capitation e-visit compensation). The second set of results relate to case b): Proposition 7.

Note that in our model neither the patient nor the physician is forced to adopt e-visits. The patients make a choice based on utility considerations as described in Proposition 2, and the physician only adopts e-visits if it increases her revenue. Therefore, if e-visits are detrimental to the physician, she will stop offering them.

### 4.2. The Impact of E-Visits on System Outcomes

In this section, we characterize the changes in the three outcomes of interest—physician revenue, panel size, and panel health—resulting from the introduction of e-visits in primary care. The expected daily revenue as well as the overall health of the patient panel in the physician’s care are key performance indicators for the physician and patients. The changes in the size of patient panel is an important indicator of overall primary care coverage that a given number of primary care physicians provides. Thus, these outcomes of interest reflect the potential attractiveness of e-visits to three key groups of stakeholders: physicians, patients, and the social planner.

We focus on a homogeneous, flexible patient panel with
\[ q \in [q^-(c, \Delta), q^+(c, \Delta)] , \] (48)
and a physician with “proportional” compensation for routine and urgent in-office visits and a mixture of fee-for-service and capitation compensations modes:
\[ \frac{R^r}{\tau^r} = \frac{R^u}{\tau^u} , \quad 0 < \delta < 1 . \] (49)
Note that in this section we focus on a homogeneous patient panel to be able to maintain analytical tractability and to provide sharper insights. We consider two possible scenarios of e-visit incentives: “proportional fee-for-service e-visit compensation” and “capitation e-visit compensation”. Under proportional fee-for-service e-visit compensation, the physician is only paid per e-visit and the e-visit compensation is proportional to the duration of e-visit, i.e., \( \frac{R_r}{\tau_r} = \frac{R_u}{\tau_u} = \frac{R_e}{\tau_e} \) and \( R_d^e = 0 \). Under capitation e-visits compensation, the physician is not paid for each e-visit, but receives a daily capitation payment per patient for providing e-visits, i.e., \( R_e^d = 0 \), \( R_d^e > 0 \).

Proposition 5 (Proportional fee-for-service e-visit compensation). Consider the setting described by (29), (48), and (49) under “proportional fee-for-service e-visit compensation” \( (\frac{R_r}{\tau_r} = \frac{R_u}{\tau_u} = \frac{R_e}{\tau_e}, R_d^e = 0) \). Noting that in such a setting patients are guaranteed to adopt e-visits, suppose that patients stay flexible after e-visit adoption, i.e., \( \alpha_r^e \leq \bar{\alpha}_r^e (q) \). Then, the introduction of e-visits produces the following effects:

a) Panel health improves if \( \bar{q}_e^r \leq q \leq q^r \), and remains unchanged otherwise.

b) Panel size increases.

c) Physician revenue increases.

Proposition 5 describes the impact of e-visits on system outcomes when the physician is paid a proportional rate for e-visits, so her revenue per visit duration is the same for all types of visits (routine in-office, routine e-visit, and urgent in-office). The proposition also focuses on a setting in which patients are flexible both before and after e-visits are introduced. Note that since \( R_d^e = 0 \) in Proposition 5, patients always opt to adopt e-visits because the long-run average cost is guaranteed to be lower with e-visits. The first insight from Proposition 5 is that, panel health is unaffected by e-visits if the panel is comprised of either rather sick or very healthy patients. Patient health, however, will improve for moderately healthy panels. The range of values for which panel health improves is between \( q^r \) and \( \bar{q}_e^r \). In this range of \( q \), the physician assigns an RVI of \( T_l \) after e-visits, while before e-visits, the physician’s choice of RVI was \( T_h \). Changes in panel health outlined in Proposition 5 are caused exclusively by physician incentives, since, in this particular setting, e-visits do not alter the degree of patient flexibility.

Both panel size and physician revenue increase when a proportional fee-for-service approach is used for compensating e-visit care provided to flexible patients. In this setting, e-visits are on equal footing with other visit types in terms of revenue and are time-saving for the physician. As a result, the physician uses e-visits to expand her panel size and earn more revenue.

For further analysis, it is convenient to introduce the following notation:

\[
\hat{R}_e^d = \left( \frac{\delta}{1-\delta} \right) \left( \frac{1-\alpha_e^r \beta_r^e}{T_l} \right) \left( \frac{q_e^r}{q^r} - \frac{1}{1-\frac{q_e^r}{q^r}} \right) - \frac{R_d^e}{R^e}, \tag{50}
\]
Consider a setting where patients choose to adopt e-visits. Under “capitation e-visit compensation” \((R^e_c = 0, R^d_c > 0)\) and the conditions described by (29), (48), and (49), suppose that patients stay flexible after e-visits are introduced, i.e., \(\alpha^e_c \leq \tilde{\alpha}^e_c(q)\). Then, the introduction of e-visits produces the following effects:

a) Panel health decreases if \((q, R^d_c) \in \Xi_{0,1}\), improves if \((q, R^d_c) \in \Xi_{1,0}\), and remains unchanged otherwise.

b) Panel size decreases if and only if

\[
(1 + \Sigma_e)^+ \leq \frac{1 - (1 - q^e_c) \alpha^e_c (\beta^e_c - \frac{\tau^e}{\tau^e})}{1 - (1 - Q^e) \alpha^e_c (\beta^e_c - \frac{\tau^e}{\tau^e})}, \quad q^e \leq q \leq \frac{\bar{q}^e_0 (1 + \Sigma_e)^+}{1 + q^0 (Q^e - q)}, \quad \frac{R^d_c}{R^c} < \tilde{R}^d_c.
\]  

(55)

c) Physician revenue decreases if and only if

\[
\left\{ q < \min \left\{ \frac{\bar{q}^e_0}{(1 + \Sigma_e)^+}, q^e \right\}, \quad G(1, 1) < 0 \right\} \cup \left\{ q > \max \left\{ \frac{\bar{q}^e_0}{(1 + \Sigma_e)^+}, q^e \right\}, \quad G(0, 0) < 0 \right\}
\]

\[
\cup \left\{ (q, R^d_c) \in \Xi_{0,1}, \quad G(0, 1) < 0 \right\} \cup \left\{ (q, R^d_c) \in \Xi_{1,0}, \quad G(1, 0) < 0 \right\}.
\]  

(56)

Similar to the previous Proposition, Proposition 6 considers a setting in which patients are flexible both before and after the introduction of e-visits. In the present setting, however, the physician is compensated for providing e-visits on a capitation basis.

Compared to the setting without e-visits, panel health stays the same if patients are either very healthy or rather sick. If patient health level is in the intermediate range, however, panel health may improve or deteriorate, depending on the amount of capitation payment for e-visits. In particular, panel health improves (deteriorates) for sufficiently large (small) values of the e-visit capitation payment. For sufficiently small e-visit capitation payments, the physician changes the RVI value from \(T_i\) to \(T_h\) upon the introduction of e-visits. A capitation-only e-visit compensation changes the balance of physician incentives from fee-for-service \((R^r, R^a, R^c)\) and capitation \((R^d, R^d_c)\) payments.

We explain the intuition behind this result with a simple example. Suppose (1) \(q > \bar{q}^e\), and (2) e-visits take almost as long as office visits, i.e., \(\tau^e \rightarrow \tau^r\); in such a case, e-visits provide no
time-saving advantage to the physician, and \( \bar{q}^* \rightarrow \bar{q}^r \). Recall that, in the absence of e-visits and under proportional compensation of office visits, the physician chooses an RVI of \( T_l \) for \( q > \bar{q}^r \) (Corollary 1). After the introduction of e-visits, if the physician keeps the RVI value at \( T_l \), her fee-for-service revenue decreases because \( R^*_e = 0 \). The capitation revenue, however, increases since \( R^d_e > 0 \) with no change in panel size. The physician can shift a higher fraction of office visits to the urgent category and increase the fee-for-service revenue by increasing the RVI values to \( T_h \), but that is guaranteed to decreases the capitation revenue as the panel size will decrease (Corollary 1). Also, note that the higher the value of \( q \), the larger the decrease in panel size if the RVI value increases. Therefore, the physician’s optimal choice of RVI depends on the value \( q \) and the capitation payments as described in (52) and (53). In particular, for sufficiently small \( q \) and e-visit capitation payment, it is optimal for the physician to increase the RVI value to \( T_h \).

Panel size may decrease if patient health level is in the intermediate range and e-visit capitation payment is small. In this case, the physician uses high RVI values to divert care from routine appointments to urgent ones to earn more revenue because e-visits provide no per-visit compensation to the physician.

Proposition 6 describes the impact of e-visits in settings where patients that are flexible in the absence of e-visits remain flexible after e-visits are introduced. Below we analyze settings where patient flexibility changes upon the introduction of e-visits. In our analysis, we use the following notation:

\[
\bar{q}^\alpha = \bar{q}^r \left( \frac{1 - Q^T \alpha^r_e (\beta^r_e - \frac{\tau^r}{\tau})}{1 - \bar{q}^r \alpha^r_e (\beta^r_e - \frac{\tau^r}{\tau})} \right).
\]  

(57)

**Proposition 7 (Impact of e-visits when patient flexibility changes).** Consider a setting where patients choose to adopt e-visits. Under the conditions described by (48) and (49), suppose that patients are not flexible after e-visits are introduced, i.e., \( \alpha^e_r > \bar{\alpha}^e_r (q) \). Then, the introduction of e-visits produces the following effects:

a) Panel health improves if \( q \leq \bar{q}^r \), and remains unchanged otherwise.

b) Panel size decreases if and only if \( q \leq \bar{q}^\alpha \).

c) Physician revenue increases under proportional fee-for-service compensation for e-visits if and only if \( q > \bar{q}^\alpha \). Physician revenue decreases under capitation compensation for e-visits if and only if

\[
\{q \leq \bar{q}^r, G(1,0) < 0\} \cup \{q > \bar{q}^r, G(0,0) < 0\}.
\]  

(58)

In the settings described by Proposition 7, patients accept both RVI values, \( T_l \) and \( T_h \), in the absence of e-visits, while insisting on \( T_l \) upon the introduction of e-visits. Changes in patient flexibility have no effect on RVI values when \( q > \bar{q}^r \) because, in the absence of e-visits, for those
value of $q$ the RVI value that maximizes physician revenue is also $T_l$. When $q \leq \bar{q}$, however, the physician chooses the RVI of $T_h$ before e-visits are introduced, but patients become inflexible with RVI equal to $T_l$ when they adopt e-visits. Overall, Proposition 7 shows that changes in RVI values as a result of patients becoming inflexible under e-visits will improve patient health but may make e-visits unsustainable as they do not encourage physician participation by negatively impacting physician revenue and panel size.

5. Numerical Results

In this section, we illustrate our theoretical results in Propositions 5 and 6. We use proportional compensation for office visits (both routine and urgent) and show how the capitation and fee-for-service elements of e-visit compensation affect system outcomes. In Figures 3-5, we highlight three possible effects: (1) e-visits increasing patient health, (2) e-visits decreasing panel health, and (3) e-visits decreasing panel size. Additionally, we discuss the patient welfare implication of e-visits in Figure 6.

In terms of e-visit compensation parameters for the numerical analysis, we use $50$ as the maximum value for $R_{re}$ because the fee-for-service e-visit compensation is currently in the $20$-$50$ range (MedInfoTech 2012). For the e-visit capitation payment, we use $0.016$ as the maximum value of $R_{de}$. Reijonsaari et al. (2005) study a health system that charges a $60$ annual fee for e-visits with $10\%$ patient adoption. The daily capitation payment for such a system is about $0.016$.

5.1. The Impact of E-Visits Panel Size and Panel Health

Note that in Figures 3-5, we ignore the hatched areas as they represent the set of parameters that result in no e-visit adoption (through either patients or physician). As the patient is the first mover, we first examine whether the patients adopt e-visits, and then consider whether the physician also adopts. Patients will not adopt e-visits when the e-visit fees are sufficiently large, as discussed in earlier Proposition 2 and Figure 1. The physician will not adopt e-visits if the compensation is sufficiently low. In what follows we focus on the “feasible” region in which both the patients and physician adopt e-visits.

We begin with Figure 3, which demonstrates an example in which patient health improves for a certain set of fee-for-service ($R_{re}$) and capitation ($R_{de}$) e-visit payments. Figures 3a and 3b show the impact of e-visits on panel size and panel health, respectively. Panel health is driven by the RVI values, so the areas in which RVI values change from $T_h$ to $T_l$ are the ones with improved patient health. One of the insights from this figure is that reducing e-visit compensation can eliminate gains in panel health under e-visits. For example, consider the following two points: $(R_{re}, R_{de}) = (20, 0.01)$

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Calculated as $\frac{60 \times 0.1}{365} = 0.016$. 

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Figure 3: Changes in system outcomes upon the introduction of e-visits as a function of fee-for-service, $R_e^c$, and capitation, $R_e^d$, e-visit payments ($q = 0.49, \delta = 0.25, c = 2, c_0 = 50, \Delta = 0.9, T_h = 120, T_l = 60, \tau^v = 1, \tau^u = 2, \tau_e^r = 0.2, R_r^e = 200, R_u^e = 400, R_r^d = 0.1, \alpha_e^r = 0.5, \beta_e^r = 1$).

and $(R_e^c, R_e^d) = (30, 0.01)$. Both points have the same e-visit capitation compensation, but the first point has smaller e-visit fee-for-service compensation. Larger e-visit compensation in the second point provides the physician with enough incentives to reduce the RVIs even though the fee-for-service e-visit compensation is less than the proportional value of $40$. In particular, because e-visits have shorter duration, shifting patient demand to routine visits by reducing the RVIs leads to a larger panel size and more capitation payments which in turn increase the physician’s revenue.

In terms of the connection between Figure 3 and the analytical results, note that $q = 0.49$ in the Figure. This value is chosen such that it is between $\bar{q}^\tau = 0.5$ and $\bar{q}^\nu_e = 0.3$, the measures of heterogeneity in the time that the physician has to invest in routine and urgent visits under the traditional and e-visit modes, respectively. Therefore, as stated in part a) of Proposition 5, we observe that panel health improves for proportional e-visit compensation, i.e., $(R_e^c, R_e^d) = (40, 0)$.

Figure 4 demonstrates a case where panel health decreases when e-visits are adopted. The values of $R_e^c$ and $R_e^d$ that lead to lower patient health in the Figure are not included in Proposition 6 because they represent a combination of fee-for-service and capitation e-visit compensation. The intuition that was developed in Proposition 6, however, is applicable here: for sufficiently low values of $R_e^d$ and $R_e^c$, the physician picks an RVI of $T_h$ for patients who had an RVI of $T_l$ under the traditional case. This increase in RVI values improve physician revenue by diverting patient demand from routine visits to urgent visits. Note that, under e-visits, routine visits are a

4 The proportional e-visit compensation is Figure 3 is $R_e^c = 40$, calculated as $200 \times \tau^v$. 

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combination of office and e-visits, so a less than proportional e-visit compensation\(^5\) makes routine visits less attractive for the physician compared to urgent visits (from a fee-for-service compensation standpoint). Another observation based on Figure 4 is that the points close to \((R^e_r, R^d_r) = (40, 0)\) are consistent with Proposition 5 which states that there is no change in patient health when e-visit compensation is proportional and \(0.51 = q > \bar{q} = 0.5\).

Figure 5 shows an example in which e-visits lead to smaller panel sizes.\(^6\) As expected, we observe that under proportional e-visits compensation, \((R^e_r, R^d_r) = (20, 0),\)\(^7\) and \(0.3 = q < \bar{q}^e_r = 0.41\), patient health is unchanged and panel size increases. As the fee-for-service element of e-visit compensation \((R^e_r)\) increases, however, we observe improvements in patient health and decreases in panel size. The reason for this change is that as per e-visit compensation becomes disproportionately generous, routine visits become more attractive to the physician as these visits include e-visits. Thus, the physician shifts patient demand from urgent visits to routine visits by assigning the RVI of \(T_l\) to patients that have the RVI of \(T_h\) under the traditional mode. The reduction in RVI values leads to a smaller panel size even after accounting for time savings associated with e-visits.

Note that physicians have the ability to increase their panel sizes by accepting more new patients to their panel; also, they can decrease their panel sizes by doing the opposite. For example, Bavafa

\(^5\) Similar to Figure 3, the proportional e-visit compensation in Figure 4 is $40.

\(^6\) Note that there are few changes between the parameters in Figure 5 and the ones in Figures 3 and 4. The e-visits replace a lower fraction of routine visits, \(\alpha^e_r = 0.2\), and take less time to conduct, \(\tau^e = 0.1\). Also, the patient panel is healthier, \(q = 0.3\), and the proportion of physician compensation that is on a fee-for-service basis increases to 75%.

\(^7\) Proportional e-visit compensation is calculated as \(\frac{R^e_r}{R^r} \times \tau^e = 200 \times 0.1 = 20\).
et al. (2018) show physicians accept 15% fewer new patients each month following e-visit adoption; as patients exit a physician’s panel over time, accepting fewer new patients leads to gradual reductions in panel size.

### 5.2. The Impact of E-Visits on Patient Welfare

From an individual patient’s standpoint, an RVI of $T_i$ provides the best health outcome, but health in the model holds value to patients only via costs which measure the overall burden of the disease. To speak to the patient welfare, one must also account for panel size in addition to the disease burden on individual patients on the physician’s panel. Consider a patient population of $M$ individuals that need healthcare. The physician can provide her service only to $N$ patients, and $M - N$ patients are left outside of the physician’s panel. The burden of disease for each patient on the panel is $D^o(\hat{r})$ under the traditional mode and $D^e(\hat{r}^e)$ under e-visits. The patients who are not covered by the physician ($M - \hat{N}$ and $M - \hat{N}^e$ under the traditional and e-visits modes, respectively) experience a disease burden of $\nu$. Note that the implicit assumption here is that patients are worse off by not being included in the physician’s panel, i.e., $\nu > D^o(\hat{r}) \geq D^e(\hat{r}^e)$. Thus, the total disease burden under the traditional mode is given by the following:

$$W^o = \hat{N} D^o(\hat{r}) + \nu \left( M - \hat{N} \right), \quad (59)$$

and, similarly, the total disease burden under e-visits is given by

$$W^e = \hat{N}^e D^e(\hat{r}^e) + \nu \left( M - \hat{N}^e \right). \quad (60)$$
We define patient welfare in terms of disease burden: the lower the disease burden, the higher is patient welfare. Thus, the change in total patient disease burden or welfare under e-visits is

\[ W^o - W^e = \hat{N} \left( D^o(\hat{r}) - \nu \right) - \hat{N}^e \left( D^e(\hat{r}^e) - \nu \right). \]  

(61)

Figure 6 provides insights related to changes in patient welfare after e-visits are introduced. The y-axis in the plot is \( \frac{W^o - W^e}{W^o} \times 100 \), the percent change in the disease burden due to e-visits. Positive values of this measure map to improved patient welfare under e-visits because they mean that total disease burden decreases when e-visits are introduced. To illustrate patient welfare implications, we use the same parameters in Figure 5 because panel size can increase or decrease depending on the value of e-visit fee-for-service payment, \( R^e \). Similar to Figure 5, in Figure 6, panel size decrease under e-visits for \( R^e > 24 \) and increases otherwise.

We observe that when e-visits increase panel size, patient welfare increases. There are two forces involved in this observation. First, patients adopt e-visits because it reduces their cost of care (Proposition 2), so, holding panel size constant, patient welfare increases. Second, when panel size increases, more patients are covered by the physician’s services, so the population’s disease burden is reduced. When panel size decreases under e-visits, however, there is loss in patient welfare. In this case, although e-visits reduce the burden of disease for each individual patient on the panel, the welfare loss from decreased patient coverage leads to loss in the overall patient welfare.
6. Conclusion
The US health care system is facing challenging times as an increasing fraction of population receives care, and the government and private insurers experiment with new approaches for compensating care providers. To control costs and provide care for a larger number of patients, the primary care system may have to augment the traditional care delivery mode with other approaches such as e-visits. Both patients and physicians are likely to adjust their behavior in the presence of these new approaches, impacting critical system metrics such as patient panel size and office revisit intervals. Understanding these changes is crucial for designing effective policies that aid a safe transition in primary care without compromising patient health or physician panel size.

Our study addresses the complexity of physician and patient interactions under different modes of primary care delivery. In our model, patients respond to changes in the way care is delivered by adjusting the range of office revisit interval values they are willing to accept. On the physician side, we consider fee-for-service and capitation compensation schemes, and model the physician’s choice of patient panel size and office revisit intervals consistent with patient preferences. We characterize the optimal RVI and panel size values with and without e-visits, and show how it impacts panel health, panel size, and physician earnings. We illustrate the resulting outcomes via numerical analysis for a range of plausible model parameters. These analyses help us characterize the impact of e-visits on the quality of provided care and on physician time savings, which enable the physician to expand her panel size or see patients more frequently.

We show that patient and physician responses to the changes in primary care delivery influence the magnitude and even the direction of changes in system outcomes. A key focus in our study is on the pricing (physician compensation) of e-visits, which is a topical question as healthcare systems attempt to price them accurately. Our work yields several insights for system outcomes depending on how e-visits are compensated relative to office visits. The main conclusion from our exercise is that healthcare systems should attempt to match the revenue rates on e-visits and office visits as much as possible (e.g., ensuring both channels provide a similar “per minute” revenue to the physician) so as to avoid distorted incentives. We show that if the compensation of e-visits, as well as routine and urgent visits, is proportional to their duration, physician revenue and panel size increase, and panel health either improves or remains unchanged. In contrast, if physician e-visit compensation has large enough deviations from proportional compensation, all of the three mentioned outcomes may suffer. In particular, we provide evidence for two such scenarios in the paper. On one hand, if the fee-for-service element of e-visit compensation is not sufficiently high, the physician increases patient RVIs to divert patient demand away from routine visits (because e-visits reduce the physician revenue from routine visits). This will lead to lower patient health. On the other hand, if physician compensation for e-visits is sufficiently above the proportional level,
the physician will decrease patient RVIs to benefit from the generously compensated e-visits. This will improve patient health but may lead to lower panel size.

We model the patients’ joint decision on e-visit adoption and the value of RVI. We show that e-visit adoption is more appealing to less healthy patients, which has important implications for how this channel of care affects system outcomes. We also show that healthier patients will only adopt e-visits if this channel can substitute a large enough portion of office visits, which is relevant in considering e-visit design. We highlight insights for patients with an intermediate health level. For these patients, if e-visits are not effective enough in terms of replacing office visits, they do not adopt e-visits; alternatively, if e-visits are too effective in replacing office visits, patients adopt e-visits but become inflexible with respect to the RVI values, demanding frequent office visits. Note that changes in patient flexibility under e-visits will not hurt patient health, but they can negatively impact panel size and physician revenue.

Our analysis focuses on practice-oriented recommendations that are based on a limited number of easy-to-estimate parameters. As a result, it relies on several simplifying assumptions. In our model, a patient that falls sick receives same-day access to treatment, and there is no backlog of patient appointments. The key feature that our modeling captures is the fact that the physician must provide buffer capacity for urgent visits; this buffer capacity could either be for the same day or backlogs. This assumption allows for closed-form characterization of system outcomes. While the “open access” model has been gaining a wider acceptance in recent years, appointment backlogs are very common in practice. The existence of backlog will be important if we consider wait sensitivity of patients or deterioration of conditions as a result of the wait. Thus, one potential extension to our work would be a model that is at the tactical level and includes the analysis of alternative care delivery modes in the presence of backlogs.

Our analysis relies on an open-loop model which assumes pre-set RVI values that are not adjusted dynamically. A natural extension to our approach would be dynamic adjustment of RVIs based on information gathered by the physician regarding the patient health state between office visits. From that point of view, e-visits are not only a channel for care flow, but also for information flow that can dynamically affect RVI values.

Actual patient health is best considered as a multi-dimensional vector that includes chronic conditions (e.g., diabetes, hypertension) and the urgent needs (e.g., skin rash). In our model, we reduce these to a one-dimensional scalar which represents an average over all conditions. In a model that captures these heterogeneous health conditions, the realization of an office visit would depend on the type of health failure; for example, a patient may have his blood pressure under control but not his diabetes. Additionally, the reality of primary care includes patients with different values of $T_1$ and $T_h$, and the value of patient health parameter, $q$, can change dynamically. These are
limitations of our model, and we made these simplifications to focus on building a model from a macroscopic view that can capture the key driving forces of outcomes such as panel size and physician compensation. We believe that although our approach is quantitatively not accurate, it is informative from a qualitative standpoint.

Finally, we characterize physician compensation using a mix of two standard forms in current practice, fee-for-service and capitation payments. While significant changes to the physician compensation structure may not be likely in the short run, there has been interest in performance-based incentives that reward physicians for improvements in the quality of patient care. Future research could examine the impact of such incentives on system outcomes in primary care.

References


Wishner, J.B., R. Burton. 2017. How have providers responded to the increased demand for health care under the affordable care act. *US Health Reform Monitoring Impact*.


Online Appendix for “Customizing Primary Care Delivery Using E-Visits”

Appendix A: Proofs of Analytical Results

Proof of Lemma 1

We use the standard renewal process framework to calculate the patient’s long-run average cost. Consider the following counting process for the number of patient visits in the interval $[0,t]$:

$$A(t) = \max \left\{ n : \sum_{j=1}^{n} \min \{ T_j, r \} \leq t \right\}, \tag{A1}$$

where $T_j$ is the $j^{th}$ value of $T$ in the $[0,t]$ interval.

The reward (cost in the units of $c_o$) earned by the patient until time $t$ is given by

$$R(t) = \sum_{j=1}^{A(t)} \mathbb{I}_{\{ T_j \geq r \}} + \mathbb{I}_{\{ T_j < r \}} (1 + \eta). \tag{A2}$$

Then, the patient’s long-run average cost is the following:

$$D_o(r) = \lim_{t \to \infty} \frac{R(t)}{t} = \frac{1}{T(r)} C(r) = \begin{cases} \frac{1}{T_l}, & r \leq T_l, \\ \frac{1 + q \eta}{q T_l + (1 - q) T_h}, & T_l < r \leq T_h, \\ \frac{1 + q \eta}{1 + q T_l + (1 - q) T_h}, & r > T_h, \end{cases} \tag{A3}$$

where we used the standard result for renewal reward processes in (A3) (Ross 1996).

Under the two-scenario distribution (1) the RVI value that minimizes (A3) is either $T_l$ or $T_h$. Since (A3) is a decreasing function of $r$ for $r \leq T_l$ and for $r \in (T_l, T_h]$, the global minimum of $D_o(r)$ is either $T_l$ or $T_h$. Comparing $D_o(T_l)$ and $D_o(T_h)$, we obtain the desired result. The minimizer is $T_l$ if $q$ is such that

$$\frac{1}{T_l} < \frac{1 + q \eta}{q T_l + (1 - q) T_h}. \tag{A4}$$

Solving (A4) for $q$, we have

$$q > \frac{1}{1 + \frac{\eta}{T_l - 1}}. \tag{A5}$$

Proof of Lemma 2

Note that $q^- (c, \Delta) = \frac{1}{1 + \frac{\Delta}{T_l - 1}} < \frac{1}{1 + \frac{\Delta}{T_h - 1}} = q^+ (c, \Delta)$ for $\Delta > 0$. Then, the value of $q$ can fall in three possible intervals:

1) $q < q^- (c, \Delta)$: using Lemma 1, $T_h$ is the minimizer of patient cost in (5) for both $\eta = c(1 - \Delta)$ and $\eta = c(1 + \Delta)$.

2) $q > q^+ (c, \Delta)$: using Lemma 1, $T_l$ is the minimizer of patient cost in (5) for both $\eta = c(1 - \Delta)$ and $\eta = c(1 + \Delta)$.

3) $q^- (c, \Delta) \leq q \leq q^+ (c, \Delta)$: using Lemma 1, $T_l$ is the minimizer of patient cost in (5) for $\eta = c(1 - \Delta)$, and $T_h$ is the minimizer of patient cost in (5) for $\eta = c(1 + \Delta)$. Therefore, the patients are willing to accept both $T_l$ and $T_h$.

□
Proof of Proposition 1
We prove a more general version of this proposition for a heterogeneous patient population with two patient classes in Proposition B2.

Proof of Corollary 1
We prove a more general version of this Corollary for a heterogeneous patient population with two patient classes in Corollary B1.

Proof of Proposition 2
There are four possible solutions for \((\bar{\theta}, \bar{r}_e)\): \((0, T_l)\), \((0, T_h)\), \((1, T_l)\), and \((1, T_h)\). We derive the necessary and sufficient conditions for each of these four options to be the optimal solution. Based (5) and (31), the long-run average cost values for the patient at each of these solutions are

\[
D(0, T_l) = \frac{1}{T_l}, \tag{A6}
\]

\[
D(0, T_h) = \frac{1 + q\eta}{qT_l + (1-q)T_h}, \tag{A7}
\]

\[
D(1, T_l) = R_c^d + \frac{1 - \left(\beta_e^r - \frac{R_r^e}{\alpha_e^r}\right)\alpha_e^r}{T_l}, \tag{A8}
\]

\[
D(1, T_h) = R_c^d + \frac{(1-q) \left(1 - \left(\beta_e^r - \frac{R_r^e}{\alpha_e^r}\right)\alpha_e^r\right) + q(1+\eta)}{qT_l + (1-q)T_h}. \tag{A9}
\]

In what follows we derive the optimality conditions for each of the aforementioned solutions. For each solution, we derive conditions under which the patient’s long-run average cost is smaller under that solution compared to the other three.

Case 1: \((\bar{\theta}, \bar{r}_e) = (0, T_l)\)

For \(D(0, T_l) < D(0, T_h)\) to hold, based on Lemma 1, we need

\[
q > \frac{1}{1 + \frac{\eta}{T_l} - 1}. \tag{A10}
\]

Next, we derive conditions for \(D(0, T_l) < D(1, T_l)\) and \(D(0, T_l) < D(1, T_h)\) to hold. We first compare the values of \(D(1, T_l)\) and \(D(1, T_h)\). This problem is identical to seeking the optimal RVI value that minimizes (31). By Comparing \(D^e(T_l)\) and \(D^e(T_h)\), we see the following:

(a) \(D(1, T_l) < D(1, T_h)\) if

\[
q > \frac{1}{1 + \frac{\eta}{T_l} - 1}. \tag{A11}
\]

(b) \(D(1, T_l) \geq D(1, T_h)\) if

\[
q \leq \frac{1}{1 + \frac{\eta}{T_l} - 1}. \tag{A12}
\]
where $\eta_0^*$ is given by (35). Note that $\eta_0^* > \eta$ for $\alpha^*_r > 0$. Therefore, (A10) guarantees (A11), and we only need to find conditions under which $D(0, T_l) < D(1, T_l)$:

$$\frac{1}{T_l} < R^d + \frac{1 - \left(\beta^*_r - \frac{R^d}{T_l} \right) \alpha^*_r}{T_l},$$

which is simplified to

$$R^d > \frac{\left(\beta^*_r - \frac{R^d}{\alpha^*_r} \right) \alpha^*_r}{T_l}.$$  \hfill (A14)

In summary, for Case 1 we need (A10) and (A14).

Case 2: $(\bar{\theta}, \bar{\rho}^*) = (0, T_h)$

For $D(0, T_h) \leq D(0, T_l)$ to hold, based on Lemma 1, we need

$$q \leq \frac{1}{1 + \frac{n_2}{\alpha^*_r - 1}}.$$  \hfill (A15)

For $D(0, T_h) < D(1, T_l)$ and $D(0, T_h) < D(1, T_h)$, there are two cases to consider as both (A11) and (A12) may hold.

If (A11) holds, we have

$$\frac{1}{1 + \frac{\eta^*}{\alpha^*_r - 1}} < q \leq \frac{1}{1 + \frac{\eta}{\alpha^*_r - 1}}.$$  \hfill (A16)

Then, we need to find conditions for $D(0, T_h) < D(1, T_l)$:

$$\frac{1 + qn_2}{qT_l + (1 - q)T_h} < R^d + \frac{1 - \left(\beta^*_r - \frac{R^d}{\alpha^*_r} \right) \alpha^*_r}{T_l},$$

which is simplified to

$$R^d > \frac{1 + qn_2}{qT_l + (1 - q)T_h} - \frac{1 - \left(\beta^*_r - \frac{R^d}{\alpha^*_r} \right) \alpha^*_r}{T_l}.$$  \hfill (A17)

If (A12) holds, then we need to find conditions for $D(0, T_h) < D(1, T_h)$:

$$\frac{1 + qn_2}{qT_l + (1 - q)T_h} < R^d + \frac{(1 - q) \left(1 - \left(\beta^*_r - \frac{R^d}{\alpha^*_r} \right) \alpha^*_r \right) + q(1 + \eta)}{qT_l + (1 - q)T_h},$$

which is simplified to

$$R^d > \frac{(1 - q) \left(\beta^*_r - \frac{R^d}{\alpha^*_r} \right) \alpha^*_r}{qT_l + (1 - q)T_h}.$$  \hfill (A18)

In summary, for Case 2 we need either of the following two to hold: both (A16) and (A18), or both (A12) and (A20).

Case 3: $(\bar{\theta}, \bar{\rho}^*) = (1, T_l)$

For $D(1, T_l) < D(1, T_h)$ to hold, we need (A11) to hold.

For $D(1, T_l) \leq D(0, T_l)$ and $D(1, T_l) \leq D(0, T_h)$ to hold, we need to consider two cases as both (A10) and (A15) may hold.

If (A10) holds, then we need to find conditions for $D(1, T_l) \leq D(0, T_l)$ which is guaranteed by (similar to (A13)-(A14))

$$R^d \leq \frac{\left(\beta^*_r - \frac{R^d}{\alpha^*_r} \right) \alpha^*_r}{T_l}.$$  \hfill (A21)
If (A15) holds, combined with (A11), then (A16) holds. Therefore, we need to find conditions for \( D(1, T_i) \leq D(0, T_h) \). Similar to (A17)-(A18), we have \( D(1, T_i) \leq D(0, T_h) \) if

\[
R_e^d \leq \frac{1 + q\eta}{qT_i + (1 - q)T_h} - \frac{1 - (\beta_e^c - \frac{R_e^c}{c_0})\alpha_e^c}{T_i}.
\]  
(A22)

In summary, for Case 3 we need either of the following two to hold: both (A10) and (A21), or both (A16) and (A22).

Case 4: \((\bar{\theta}, \bar{r}) = (1, T_h)\)

For \( D(1, T_h) \leq D(1, T_i) \), we need (A12) to hold. Also, if (A12) holds, we have \( D(0, T_h) < D(0, T_i) \) because (A12) guarantees (A15). Therefore, we only need to provide conditions for \( D(1, T_h) \leq D(0, T_h) \) which is guaranteed by (similar to (A19)-(A20))

\[
R_e^d \leq \frac{(1 - q)(\beta_e^c - \frac{R_e^c}{c_0})\alpha_e^c}{qT_i + (1 - q)T_h}.
\]  
(A23)

In summary, for Case 4 we need (A12) and (A23).

\[ \Box \]

**Proof of Proposition 3**

Suppose that we are in a setting where patients choose to adopt e-visits. Note that, for \( \alpha_e^c > 0 \), we have \( q^+(c, \Delta, R_e^c, \alpha_e^c) < q^+(c, \Delta) \) and \( q^-(c, \Delta, R_e^c, \alpha_e^c) < q^-(c, \Delta) \).

a) Consider a patient panel that is flexible in the absence of e-visits, so that \( q^-(c, \Delta) \leq q \leq q^+(c, \Delta) \). In order for a flexible group \( i \) to remain flexible upon the introduction of e-visits, it is necessary to have \( q \leq q_e^+(c, \Delta, R_e^c, \alpha_e^c, \beta_e^c) \) which is equivalent to \( \alpha_e^c \leq \bar{\alpha}_e^c(q) \).

b) A flexible patient panel becomes inflexible upon the introduction of e-visits if and only if \( q_e^+(c, \Delta, R_e^c, \alpha_e^c, \beta_e^c) < q \):

\[
\frac{1}{1 + \frac{c(1 - \Delta) + (\beta_e^c - \frac{R_e^c}{c_0})\alpha_e^c}{(1 - (\beta_e^c - \frac{R_e^c}{c_0})\alpha_e^c)(\frac{1}{q} - 1)}} < q,
\]
(A24)

which is equivalent to \( \alpha_e^c > \bar{\alpha}_e^c(q) \).

c) Given \( q_e^+(c, \Delta, R_e^c, \alpha_e^c, \beta_e^c) < q^+(c, \Delta) \) and \( q_e^-(c, \Delta, R_e^c, \alpha_e^c, \beta_e^c) < q^-(c, \Delta) \), the only possible scenario under which an inflexible patient group becomes flexible is when \( q \leq q^-(c, \Delta) \) and \( q_e^-(c, \Delta, R_e^c, \alpha_e^c, \beta_e^c) \leq q \leq q_e^+(c, \Delta, R_e^c, \alpha_e^c, \beta_e^c) \). Note that

\[
q \leq q_e^+(c, \Delta, R_e^c, \alpha_e^c, \beta_e^c),
\]
(A25)

is equivalent to

\[
(\beta_e^c - \frac{R_e^c}{c_0})\alpha_e^c \leq \frac{\left(\frac{1}{q} - 1\right)\left(\frac{T_h}{l_1} - 1\right) - c(1 - \Delta)}{\left(\frac{1}{q} - 1\right)\left(\frac{T_h}{l_1} - 1\right) + 1},
\]
(A26)

and

\[
q \geq q_e^-(c, \Delta, R_e^c, \alpha_e^c),
\]
(A27)

is equivalent to

\[
(\beta_e^c - \frac{R_e^c}{c_0})\alpha_e^c \geq \frac{\left(\frac{1}{q} - 1\right)\left(\frac{T_h}{l_1} - 1\right) - c(1 + \Delta)}{\left(\frac{1}{q} - 1\right)\left(\frac{T_h}{l_1} - 1\right) + 1}.
\]
(A28)

Solve both of the above inequalities results in \( \alpha_e^c(q) \leq \alpha_e^c \leq \bar{\alpha}_e^c(q) \).

\[ \Box \]
Proof of Proposition 4
We prove a more general version of this proposition for a heterogeneous patient population with two patient classes in Proposition B4.

Proof of Proposition 5
Note that in this case \( \bar{q}^* > q^* \).

a) Using Propositions B2 and B4, we know that, before e-visits are introduced, \( \hat{\tau} = T_i \) if \( q > \bar{q}^* \), and given that \( \bar{q}^* > q^* \), the RVI stays at \( T_i \) for \( q > \bar{q}^* \) upon the introduction of e-visits. Similarly, \( \hat{\tau} = T_h \) if \( q \leq q^* \), and, given that \( \bar{q}^* > q^* \), the RVI stays at \( T_h \) for \( q \leq q^* \) after e-visits are introduced. If \( q^* \geq q > \bar{q}^* \), however, the value of RVI changes from \( T_h \) to \( T_i \).

b) Note that from (39) we have

\[
\hat{N}^e = \frac{A}{\rho'(\tau)(1-\rho')\tau}. \tag{A29}
\]

Also, \( \bar{\tau}e < \tau^e \). Therefore, if \( T(\tau) \) does not change after e-visits are introduced, \( \hat{N}^e > \hat{N} \). That is, if RVI does not change upon the introduction of e-visits, panel size is guaranteed to increase. Therefore, panel size increases if \( q \leq q^* \), or if \( q > \bar{q}^* \) because \( \hat{\tau} = \hat{\tau}^e \).

From part a), \( \hat{\tau} < \hat{\tau}^e \) if \( \bar{q}^* \leq q < q^* \), and we want to find the conditions that ensure \( \hat{N}^e \leq \hat{N} \):

\[
\hat{N}^e \leq \hat{N} \iff \frac{A}{\rho'(\tau)} \leq \frac{A}{\rho'(\tau)} \iff \frac{\bar{\tau}e}{T_i} \geq \frac{(1-q)\tau^e + q\tau^u}{qT_i + (1-q)T_h} \iff q \leq \bar{q}^u. \tag{A30}
\]

Note that

\[
\bar{q}^u \leq q^* \leq \bar{q}^e, \tag{A31}
\]

because

\[
\frac{(1-q)^e}{qT_i + (1-q)T_h} \geq \frac{(1-q)^e + q^u \tau^u}{qT_i + (1-q)T_h} \iff \frac{\bar{\tau}e}{T_i} \geq \frac{qT_i + (1-q)T_h}{qT_i + (1-q)T_h} > \frac{\bar{\tau}e}{T_i}. \tag{A32}
\]

c) We want to establish necessary and sufficient conditions for \( \Pi^e(\hat{\tau}, \hat{\tau}^e) \leq \Pi_d(\hat{N}, \hat{\tau}) \).

If \( \hat{\tau} = \hat{\tau}^e = T_i \), we need the following:

\[
(1-\delta)R^d + \frac{\bar{\tau}^e}{T_i} \leq \frac{(1-\delta)R^d + \bar{\tau}^e}{T_i} \iff \frac{\bar{\tau}^e}{T_i} \leq \frac{\bar{\tau}^e}{T_i} \iff \tau^e \geq \tau^e, \tag{A33}
\]

which contradicts \( \tau^e < \tau^e \).

If \( \hat{\tau} = \hat{\tau}^e = T_h \), we need the following:

\[
(1-\delta)R^d + \delta \frac{\bar{\tau}^u}{T_i} \leq \frac{(1-\delta)R^d + \delta \frac{\bar{\tau}^u}{T_i}}{qT_i + (1-q)T_h} \iff \tau^e \geq \tau^e, \tag{A34}
\]

which contradicts \( \tau^e < \tau^e \).

If \( \hat{\tau} > \hat{\tau}^e \), we need the following:

\[
(1-\delta)R^d + \delta \frac{\bar{\tau}^u}{T_i} \leq \frac{(1-\delta)R^d + \delta \frac{\bar{\tau}^u}{T_i}}{qT_i + (1-q)T_h} \iff \tau^e \geq \tau^e, \tag{A35}
\]

which contradicts the fact that \( \hat{\tau} < \hat{\tau}^e \) if \( q^* \leq q < \bar{q}^* \). Note that we showed \( \bar{q}^u \leq \bar{q}^e \) in (A31). □
Proof of Proposition 6

Suppose that we are in a setting where patients choose to adopt e-visits.

First, we show conditions for \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \). We know that \( q_{\tau}^e < q^r \), so to have \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \), we need \( \Sigma_{\tau} < 0 \) which is guaranteed if \( \bar{q}_{\tau}^e < q^r \). Also, note that if \( \Sigma_{\tau} \leq -1 \), then \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \). Therefore, we focus on \(-1 < \Sigma_{\tau} < 0\), and provide conditions for \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \):

\[
\frac{q_{\tau}^e}{1+\Sigma_{\tau}} > q^r \Leftrightarrow \Sigma_{\tau} < \frac{q_{\tau}^e}{q^r} - 1 \Leftrightarrow \frac{R_{\tau}^d}{R^c} < \bar{R}_{\tau}^d.
\]

Therefore, if \( \frac{q_{\tau}^d}{q^r} < \bar{R}_{\tau}^d \), then \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \).

a) Using Propositions B2 and B4, we know that, prior to the introduction of e-visits, \( \hat{r} = T_l \) if \( q > q^r \), but after e-visits are introduced, \( \hat{r} = T_l \) if \( q > \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} \). Therefore, if \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \), the RVI values increase for \( q^r < q \leq \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} \). Also, when \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \), the RVI values decrease for \( q^r < q \leq q^r \).

b) From part a), \( \hat{r} < \hat{r} \) if \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} > q^r \). Under this condition, we want to find the additional conditions ensuring that \( \hat{N}^e \leq \hat{N} \). From (A30), we only need \( q \leq q^e \). Note, however, that this interval has no overlap with \( \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} \leq q < q^r \), based on (A31).

Also, \( \hat{r}^e \leq \hat{r} \) if \( q^r < q \leq \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} \). Under this condition, we want to find the additional conditions ensuring that \( \hat{N}^e \leq \hat{N} \):

\[
\hat{N}^e \leq \hat{N} \Leftrightarrow \frac{A}{q_{\tau}^e T_l + (1-q_{\tau}^e)T_h} \leq \frac{A}{T_l} \Leftrightarrow q \geq q^e,
\]

So, panel size decreases if \( q^e < q \leq \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} \). Next, we need to ensure that this interval is not empty. First, note that

\[
\hat{q}^e \geq q^e,
\]

because

\[
\frac{(1-q^e)^{-r} + q^e \tau^u}{q_{\tau}^e T_l + (1-q_{\tau}^e)T_h} = \frac{(1-q^e)^{-r} + q^e \tau^u}{T_l} \Rightarrow \frac{1-q^e}{q_{\tau}^e T_l + (1-q_{\tau}^e)T_h} < \frac{\tau^r}{T_l}.
\]

Second, we need to have \( \hat{q}^e \leq \frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} \). That is,

\[
\frac{q_{\tau}^e}{(1+\Sigma_{\tau})^r} \geq \hat{q}^e \left( 1 - \left( 1 - Q^r \right) \alpha^r \left( \beta^e - \frac{\tau^e}{\tau^r} \right) \right) / \left( 1 - \left( 1 - Q^r \right) \alpha^r \left( \beta^e - \frac{\tau^e}{\tau^r} \right) \right)
\]

which is equivalent to

\[
(1+\Sigma_{\tau})^r \leq \frac{1 - (1-q^e)^{-r}}{1 - (1-Q^r) \alpha^r \left( \beta^e - \frac{\tau^e}{\tau^r} \right)}.
\]

c) There are four cases to consider, \( \hat{r} = \hat{r}^e = T_l \), \( \hat{r} = \hat{r}^e = T_h \), \( \hat{r} > \hat{r}^e \), and \( \hat{r} < \hat{r}^e \). Under each of these cases, we look for conditions that guarantee \( \Pi^e \left( \hat{N}^e, \hat{r}^e \right) < \Pi^e \left( \hat{N}, \hat{r} \right) \). Similar to the more general case shown in (B51):

\[
\Pi^e \left( \hat{N}, \hat{r} \right) = \frac{\hat{R} + \frac{q^e-q^r}{q^e(Q^r-q)} I}{1 + \left( \frac{q^e-q^r}{q^e(Q^r-q)} \right)}.
\]
and

\[
\Pi^e \left( \hat{N}^e, \hat{r}^e \right) = \frac{\hat{R}_e + \left( \frac{q - \bar{q}}{\bar{q} R_e (Q_T - q)} \right) I_e}{1 + \left( \frac{q - \bar{q}}{q_e (Q_T - q)} \right) I_e},
\]

(A43)

where \( I = 1 \) if \( \hat{r} = T_h \), and \( I = 0 \) otherwise. Similarly, \( I_e = 1 \) if \( \hat{r}^e = T_h \), and \( I_e = 0 \) otherwise. The difference between the two expressions in (A42) and (A43) is \( G \), so \( G < 0 \) means lower revenue for the physician under e-visits compared to the case without e-visits.

The four cases mentioned above (i.e., \( \hat{r} = \hat{r}^e = T_l \), \( \hat{r} = \hat{r}^e = T_h \), \( \hat{r} > \hat{r}^e \), and \( \hat{r} < \hat{r}^e \)) can be described in the following way: (1) if \( q < \min \left\{ \frac{q^*}{(1 + \sum_e)^T}, q^* \right\} \), we have \( \hat{r} = \hat{r}^e = T_l \), (2) if \( q > \max \left\{ \frac{q^*}{(1 + \sum_e)^T}, q^* \right\} \), we have \( \hat{r} = \hat{r}^e = T_h \), (3) if \( (q, R_e) \in \Xi_{1,0} \), we have \( \hat{r} = T_h, \hat{r}^e = T_l \), and (4) if \( (q, R_e) \in \Xi_{0,1} \), we have \( \hat{r} = T_l, \hat{r}^e = T_h \).

**Proof of Proposition 7**

We assume that we are in a setting where patients choose to adopt e-visits. Note that in the setting described by the proposition, \( \alpha^e > \bar{\alpha}^e (q) \), so \( \hat{r}^e = T_l \).

a) RVIs decrease if \( q < \bar{q}^* \) since in this case \( \hat{r} = T_h \).

b) If \( \hat{r}^e < \hat{r} \), then, as it follows from (A30), panel size decreases upon the introduction of e-visits if \( q \leq \bar{q}^* \).

Note that from (A31) we have \( \bar{q}^* < \bar{q}^\alpha \), so that \( q \leq \bar{q}^\alpha \) is a necessary and sufficient condition for panel size to decrease after e-visits are introduced.

c) For the proportional fee-for-service e-visit compensation, similar to (A35), we have \( q \leq \bar{q}^\alpha \). For the capitation e-visit compensation, we need to consider two separate cases, (1) \( \hat{r} = T_h \), and (2) \( \hat{r} = T_l \). The first case occurs when \( q < \bar{q}^* \), so that for revenue to decrease we need \( G(1,0) < 0 \). The second case occurs when \( q > \bar{q}^* \), so that for revenue to decrease we need \( G(0,0) < 0 \).
Appendix B: Heterogeneous Effects

B.1. Capacity Allocation and Physician Compensation

In this section, we consider a physician who serves a heterogeneous panel of \( N \) patients, with \( N_i = \kappa_i N \) patients belonging to the patient group \( i = 1, 2 \), and \( \kappa_1 + \kappa_2 = 1 \). The fractions \( (\kappa_1, \kappa_2) \) represent exogenous quantities that reflect the composition of the patient population in the physician’s local area. The physician may be able to select the RVI values for each patient group, \( r_i \), within the range of values acceptable to patients, but she can only control the size of each patient group, \( N_i \), by varying the panel size \( N \). This assumption reflects the reality of care delivery in the US where a physician, in general, cannot “close” her panel to any particular patient group. While it is possible for a physician to exercise an indirect control of the size of some patient groups by refusing to accept a particular kind of insurance coverage (for example, reducing the number of elderly patients by rejecting the Medicare coverage), we consider this possibility atypical and do not model it in the analysis.

Similar to (10), we assume that the physician has to provide sufficient daily appointment capacity to deal with the total expected daily demand from all patient groups:

\[
N \left( \kappa_1 \left( \frac{\rho_1^1 (r_1) \tau^r + (1 - \rho_1^1 (r_1)) \tau^u}{T_1 (r_1)} \right) + \kappa_2 \left( \frac{\rho_2^1 (r_2) \tau^r + (1 - \rho_2^1 (r_2)) \tau^u}{T_2 (r_2)} \right) \right) \leq A, \tag{B1}
\]

In choosing the size of her patient panel \( N \) and the revisit intervals \( r_i \) for each patient group, a physician is guided by her compensation scheme. Similar to what we showed in (11) and (12), the expected daily compensation for a fee-for-service physician is:

\[
\Pi_{FFS} (N, r_1, r_2) = N \left( \kappa_1 \left( \frac{\rho_1^1 (r_1) R^r + (1 - \rho_1^1 (r_1)) R^u}{T_1 (r_1)} \right) + \kappa_2 \left( \frac{\rho_2^1 (r_2) R^r + (1 - \rho_2^1 (r_2)) R^u}{T_2 (r_2)} \right) \right), \tag{B2}
\]

and under the capitation scheme, the physician is incentivized to maximize the size of her patient panel \( N \):

\[
\Pi_{CAP} (N, r_1, r_2) = N (\kappa_1 R^d + \kappa_2 R^d) = NR^d, \tag{B3}
\]

where \( R^d \) is the fixed daily compensation for each patient, whether from group 1 or group 2.

The physician’s total compensation is a combination of fee-for-service and capitation components that reflects a mix of insurance policies used by the patients on her panel:

\[
\Pi (N, r_1, r_2) = \delta \Pi_{FFS} (N, r_1, r_2) + (1 - \delta) \Pi_{CAP} (N, r_1, r_2)
\]

\[
= N \left( (1 - \delta) R^d + \delta \kappa_1 \left( \frac{\rho_1^1 (r_1) R^r + (1 - \rho_1^1 (r_1)) R^u}{T_1 (r_1)} \right) + \delta \kappa_2 \left( \frac{\rho_2^1 (r_2) R^r + (1 - \rho_2^1 (r_2)) R^u}{T_2 (r_2)} \right) \right). \tag{B4}
\]

Thus, the problem of selecting patient panel size \( N \) and the office revisit intervals \( r_1 \) and \( r_2 \) that a physician faces can be formulated as

\[
\max_{N, r_1, r_2} \Pi (N, r_1, r_2) \tag{B5}
\]

s.t. \( N \left( \kappa_1 \left( \frac{\rho_1^1 (r_1) \tau^r + (1 - \rho_1^1 (r_1)) \tau^u}{T_1 (r_1)} \right) + \kappa_2 \left( \frac{\rho_2^1 (r_2) \tau^r + (1 - \rho_2^1 (r_2)) \tau^u}{T_2 (r_2)} \right) \right) \leq A, \tag{B6}
\]

\[r_1 \in \{ r_1^-, r_1^+ \}, \quad r_2 \in \{ r_2^-, r_2^+ \}. \tag{B7}\]

We use the notation \( \{ \hat{N}, \hat{r}_1, \hat{r}_2 \} \) to denote the values of patient panel size and revisit intervals that optimize (B5)-(B7). Below we describe the values of \( \{ \hat{N}, \hat{r}_1, \hat{r}_2 \} \) for the setting with two “flexible” patient groups.
B.2. Quantifying the Impact of Revisit Interval Customization

The RVI selection process involves adapting the frequency of scheduled office visits to patient health status. For example, the physician may assign a lower RVI to “sicker” patients to check on them more frequently, while reserving less frequent office visits for “healthier” patients. In practice, the RVI values are often significantly influenced by subjective factors, in addition to objective patient characteristics (Welch et al. 1999, Schwartz et al. 1999, DeSalvo et al. 2000). In particular, Schectman et al. (2005) state that “provider practice style may reflect scheduling habits acquired from previous training independent of the patients’ medical needs. For example, providers have often been trained to schedule their patients every 3 or 4 months routinely, regardless of disease severity.” This observation underscores a potential opportunity for improving the utilization of physician care capacity by incorporating patient health status into the process of selecting the RVI values.

In this section, we analyze the benefits of adapting frequency of scheduled office visits to patient health status. In particular, we will characterize the optimal RVI decisions in the setting where both patient groups are flexible. Specifically, we consider 2 patient groups such that

\[ q_i \in [q^{-} (c, \Delta), q^{+} (c, \Delta)], i = 1, 2 \]

which is equivalent to \( r_{1i} = r_{2i} = T_{i} \) and \( r_{1i}^{+} = r_{2i}^{+} = T_{h} \). In the setting with 2 flexible patient groups, the physician can set the total panel size as well as the RVI values for both patient groups. Note that if one of the patient groups is inflexible, the problem reduces to one with a homogeneous panel and reduced appointment capacity, and if both patient groups are inflexible, the physician has no choice regarding patient RVI values.

Below we derive the expressions for the optimal panel size and the RVI values in the “heterogeneous” setting. In order to characterize the value of using different RVIs for different patient groups, we first look at the “base” case where the physician assigns the same RVI to both groups of patients and, hence, forgoes the potential advantages of customizing the RVI values to patient health. We use the term “uniform RVI” to describe this policy.

**Proposition B1 (Uniform RVI policy).** Consider a heterogeneous patient panel with \( q_i \in [q^{-} (c, \Delta), q^{+} (c, \Delta)], i = 1, 2, q_1 < q_2 \), and define

\[
\hat{\kappa}_1 = \frac{1}{1 + \left( \frac{q^{+} - q_1}{q^{+} - q_2} \right)}.
\]

(B8)

Suppose that the physician applies the same RVI value, \( \hat{r} \), to both patient groups. Then, the optimal \( \hat{r} \) in (14)-(16) is given by

\[
\hat{r} = \begin{cases} 
T_{i}, & \text{if } \frac{q^{+}}{(1+\Sigma)^{+}} < q_1 < q_2, \text{ or } q_1 \leq \frac{q^{+}}{(1+\Sigma)^{+}} < q_2 \text{ and } \kappa_1 \leq \hat{\kappa}_1, \\
T_{h}, & \text{if } q_1 < q_2 \leq \frac{q^{+}}{(1+\Sigma)^{+}}, \text{ or } q_1 \leq \frac{q^{+}}{(1+\Sigma)^{+}} < q_2 \text{ and } \kappa_1 > \hat{\kappa}_1.
\end{cases}
\]

(B9)

Proposition B1 characterizes the optimal value of the “uniform” RVI in three separate settings. In first setting \( q_1 < q_2 \leq \frac{q^{+}}{(1+\Sigma)^{+}} \), both patient groups are relatively healthy, and the optimal “uniform” RVI value is \( T_{h} \). The second setting is the opposite of the first one: when \( \frac{q^{+}}{(1+\Sigma)^{+}} < q_1 < q_2 \), both patient groups are relatively “sick”, and \( T_{i} \) becomes the optimal RVI to apply to both groups.

In the third setting, we have \( q_1 < \frac{q^{+}}{(1+\Sigma)^{+}} < q_2 \), and there is tension between the two patient groups since patients of groups 1 are relatively healthy, while patients of group 2 are relatively sick. If all patients on the
physician’s panel were of group 1, the physician would choose $T_h$, while if all patients were of group 2, the physician would choose $T_l$. If both patient groups are on the physician’s panel, and the physician chooses a single RVI for both groups, the optimal RVI depends on the value of $\hat{\kappa}_1$. In particular, the physician chooses $T_h$ if and only if the fraction of healthy patients on the panel is large enough, i.e., if and only if $\kappa_1 > \hat{\kappa}_1$. In other words, $\hat{\kappa}_1$ is the critical fraction of healthy patients on the panel beyond which the physician ignores the less healthy patients and assigns an RVI of $T_h$ to both patient groups. Figure B1 shows how this critical panel fraction value $\hat{\kappa}_1$ depends on the indicator of group 1’s health, $q_1$. As the value of $q_1$ increases, and the group 1 becomes less healthy, the physician will become increasingly inclined to apply $T_l$ to both patient groups.

We now turn our attention to the setting where the physician can assign different RVI values to the two groups of patients. We use the term “customized RVI” to describe this setting.

**Proposition B2 (Customized RVI).** Consider a heterogeneous patient panel with $q_i \in [q^- (c, \Delta), q^+ (c, \Delta)]$, $i = 1, 2$ and $q_1 < q_2$. Then, the optimal RVI values in (14)-(16) are given by

$$\hat{r}_1 = \begin{cases} T_h, & q_1 \leq \frac{q^r}{(1+\Sigma)}, \\ T_l, & \text{otherwise}, \end{cases} \quad (B10)$$

$$\hat{r}_2 = \begin{cases} T_h, & q_1 \leq \frac{q^r}{(1+\Sigma)}, q_2 \leq \frac{q^r}{(1+\Sigma)\kappa_1(1+\Sigma) + \kappa_1 \Sigma - q_1 (1+\Sigma)}, \\ T_l, & \text{otherwise}, \end{cases} \quad (B11)$$

where $x^+ = \max(x, 0)$.

The results of Proposition B2 outline the nature of the trade-offs between the revenue contributions and the capacity requirements of patient groups “competing” for the limited service capacity. Note that the choice of the RVI value for the “healthier” patient group (group 1) is affected only by the health level of patients from that group, and not by the health level of group 2 patients. In contrast, the decision of how much of
Figure B2: The optimal RVI values as functions of patient health levels \( q_1 \) and \( q_2 > q_1 \) in settings with disproportionate compensation for routine visits (case a), \( R^u = 200 \), and disproportionate compensation for urgent visits (case b), \( R^u = 230 \) (\( c = 1000, \Delta = 1, T_h = 360, T_l = 90, \tau^u = 1, \tau^u = 2.25, R^u = 100, R^d = 0.1, \kappa_1 = \delta = 0.5 \)).

the physician’s service capacity must be allocated to “sicker” patients is, in general, strongly affected by the health level of “healthier” patients. RVI decisions, however, are not affected by the amount of the daily service capacity \( A \).

Figure B2 illustrates the optimal RVI values for two patient groups in settings with \( \bar{q}^u < \bar{q}^R \) (Figure B2a) and \( \bar{q}^u > \bar{q}^R \) (Figure B2b). Note that \( \bar{q}^u < \bar{q}^R \) describes, according to (17) and (18), a setting where \( \frac{R^u}{\bar{q}^u} < \frac{R^r}{\bar{q}^r} \), i.e., a setting where routine visits are compensated disproportionately more than urgent visits, while the setting with \( \bar{q}^u > \bar{q}^R \) is the one where \( \frac{R^u}{\bar{q}^u} > \frac{R^r}{\bar{q}^r} \), and urgent visits are compensated, on a per-unit-of-time basis, better than routine visits. In Figure B2 we set \( c = 1000 \) and \( \Delta = 1 \), which results in \( q^- (c, \Delta) \approx 0.001 \) and \( q^+ (c, \Delta) = 1 \), allowing us to focus on the choices of RVI values made by a physician facing “nearly perfectly” flexible patients. In both settings, it is optimal to apply the same RVI to both patient groups if their health parameters are close in value. In particular, when both \( q_1 \) and \( q_2 \) are small, so that both patient groups are similarly “healthy”, the optimal RVI is \( T_h \) for both patient groups. In a similar fashion, when both patient groups are similarly “sick”, so that both \( q_1 \) and \( q_2 \) are high, the RVI for both patient groups are set to \( T_l \). On the other hand, if the patient population exhibits a significant degree of heterogeneity, i.e., if patient group 1 is significantly healthier than patient group 2, the physician will benefit from applying different RVI values to two patient groups, seeing group 2 patients as often as possible, and group 1 patients as infrequently as possible. It is interesting to observe that, when the group 1 patients are healthy (\( q_1 < \frac{\bar{q}^u}{(1+\Sigma)^2} \)), the threshold value of the degree of healthiness of group 2 which induces the switch in the RVI value for that group from \( T_h \) to \( T_l \), is a monotone decreasing function of \( q_1 \) for \( \Sigma > 0 \) (Figure B2a) and a monotone increasing function of \( q_1 \) for \( \Sigma < 0 \) (Figure B2b).

Thus, in settings where urgent visits are “undercompensated” compared to routine visits, the healthier the group 1 patients are, the more the physician is inclined to apply the same high revisit interval value to both patient groups. This effect is reversed in the settings where the urgent visits are “overcompensated”: the healthier the group 1 patients are, the more the physician is inclined to apply different RVIs to the two patient groups.
The intuition behind this result is the following. First, as we discussed earlier, the RVI of the healthier patient group (group 1) does not depend on the health level of the sicker patient group (group 2). Second, when urgent visits are “undercompensated”, from the standpoint of fee-for-service incentives, the physician prefers \( T_1 \) to \( T_h \) for both patient groups. Therefore, the only reason for a \( T_h \) RVI is the capitation incentive. Suppose \( q_1 \leq \frac{q^c}{1+\Sigma} \), so the optimal RVI for group 1 patients is \( T_h \). Also, note that the smaller \( q_1 \), the larger is the increase in panel size when the physician changes the RVI from \( T_1 \) to \( T_h \). The physician needs to decide on the RVI value of the second patient group weighting the fee-for-service and capitation incentives. In particular, the capitation incentives are the part where the two patient groups become connected; if panel size increases, it will increase for both patient types (recall, \( N = \kappa_1 N_1 + \kappa_2 N_2 \)). Holding \( q_2 \) constant, the smaller the value of \( q_1 \), the larger the increase in panel size if the physician changes the RVI of the second patient group from \( T_1 \) to \( T_h \). This effect leads to the threshold value of \( q_2 \) in (B11) being a monotone decreasing function of \( q_1 \) for \( \Sigma > 0 \) (Figure B2a).

Note that in both settings shown in Figure B2, we have \( \Sigma > q^c - 1 \), so that there exists a range of values of \( q_1 > \frac{q^c}{1+\Sigma} \) for which the optimal RVI value for group 1 patients is \( T_i \). Figure B3 looks at the settings with \( \Sigma < q^c - 1 \) where the urgent visits are “overcompensated” to such an extreme that it is optimal for the physician to choose an RVI of \( T_h \) for the healthier patient group regardless of \( q_1 \). That is, it could be the case that an RVI of \( T_h \) for a large \( q_1 \) leads to a reduction in panel size and capitation compensation, but this reduction is more than compensated by the increase in the fee-for-service compensation of urgent visits. The optimal RVI for the sicker patient groups depends on the degree of overcompensation. In particular, if the urgent visit “overcompensation” grows even further, such that \( \Sigma \leq (q^c - 1) \frac{q^c}{q^c - \kappa_1} \), then it becomes optimal for the physician to see patients as infrequently as possible by setting the RVI values for both patient groups at \( T_h \), irrespective of patients’ health levels (Figure B3b). If, however, \( (q^c - 1) \frac{q^c}{q^c - \kappa_1} < \Sigma < q^c - 1 \), then it is optimal assign an RVI value of \( T_i \) to the sicker patient group if \( q_2 \) is sufficiently large (Figure B3a).

Similar to Corollary 1, the expressions for the optimal RVI decisions could be simplified when the office visit compensation rates are proportional, i.e., \( R_f \frac{\tau^e}{\tau^c} = R^u \frac{\tau^u}{\tau^c} \).
Corollary B1. Consider a heterogeneous patient panel with \( q_i \in [q^{-}(c, \Delta), q^{+}(c, \Delta)] \), \( i = 1, 2 \) and \( q_1 < q_2 \) under the “proportional” compensation \( \frac{\mu_i}{\tau_i} = \frac{\mu_2}{\tau_2} \). Then, the optimal RVI values in (14)-(16) are given by

\[
\hat{r}_i = \begin{cases} 
T_l, & q_i \geq \bar{q}, \\
T_h, & q_i < \bar{q}, 
\end{cases}, i = 1, 2. \tag{B12}
\]

Corollary (B12) states that the proportional compensation for urgent and routine visits “decouples” the RVI decisions for the two patient groups, allowing each to be evaluated in isolation. The threshold value of \( q \) for switching the RVIs in (B12) is \( \bar{q}' \), which is the same threshold as the one in (23).

In what follows, we examine how RVI customization influences the three outcomes of interest: physician revenue, panel size, and panel health. As a measure of panel health, we define \( H(r_1, r_2) \) as the portion of office visits that are routine:

\[
H(r_1, r_2) = \kappa_1 \rho_1^*(r_1) + \kappa_2 \rho_2^*(r_2). \tag{B13}
\]

Note that high values of \( H(r_1, r_2) \) correspond to a well-maintained patient panel whose primary care needs are served mainly through regular visits. If both patient groups on the panel have an RVI of \( T_l \), then panel health is equal to its highest possible value, 1. Alternatively, if both patient groups have an RVI of \( T_h \), the value of panel health drops to its lowest possible value, \( \kappa_1 (1 - q_1) + \kappa_2 (1 - q_2) \).

Figure B4 takes the uniform RVI policy as the base case and shows how RVI customization can affect all three outcomes of interest. Note that in this figure \( \hat{N}^U \) represents the optimal panel size under the uniform RVI policy. We expect physician revenue to be higher in the presence of RVI customization than under the uniform RVI policy since the physician’s revenue maximization problem in former case is the relaxed version of the problem in the latter case. We also observe that the highest percentage increase in physician revenue occurs at \( \kappa_1 = \hat{\kappa}_1 \). In addition, the impact of RVI customization on physician revenue is smaller for high and low values of \( \kappa_1 \) since in those cases the patient panel is close to being homogeneous.

On the other hand, panel size may increase or decrease as a result of RVI customization depending on the fraction of healthy patients \( \kappa_1 \). Panel size increases under RVI customization compared to the uniform policy if and only if \( \kappa_1 \leq \hat{\kappa}_1 \). When the value of \( \kappa_1 \) increases beyond \( \hat{\kappa}_1 \), the optimal RVI value under the uniform RVI policy switches from \( T_l \) to \( T_h \) for both patient groups. The RVI customization approach, however, keeps the RVI value of the “sicker” patient group (group 2) at \( T_l \). The panel size under the uniform policy is thus smaller because an RVI of \( T_l \) for the “sicker” patient group leads to a surge in the number of urgent appointments which take longer to serve.

Similar to the panel size, panel health can improve or deteriorate upon the introduction of RVI customization, depending on the value of \( \kappa_1 \). For small values of \( \kappa_1 \), the physician assigns \( T_l \) to both patient groups under the “uniform RVI” policy, so that all visits are routine. When the RVI customization is used, however, healthy patients (group 1) are assigned the RVI of \( T_h \), while sick patients (group 2) are assigned the RVI of \( T_l \). Thus, for small values of \( \kappa_1 \), the overall panel health is lower under the RVI customization because the RVI values increase for patients in group 1. This effect is reversed for high values of \( \kappa_1 \) where the RVI values decrease for group 2 patients.
Figure B4: The impact of RVI customization on three key performance indicators as a function of the fraction of healthy group $\kappa_1$. The base case is the uniform RVI policy.

$(q_1 = 0.2, q_2 = 0.8, R^d = 0.1, \Delta = 0.95, T_h = 360, T_i = 90, \tau^r = 1, \tau^n = 1.5, \delta = 0.25, R^u = 220, R^r = 200)$. 

For the next proposition, we define the following notation:

$$\xi(q_i) = \frac{1 + q_i \left( \frac{1}{q_i} - 1 \right) \left( \frac{T_h}{T_i} - 1 \right)}{1 + q_i \left( \frac{1}{q_i} - 1 \right) \left( \frac{T_h}{T_i} - 1 \right)}.$$  \hfill (B14)

We also define the relative loss of revenue for the physician resulting from using the uniform RVI policy by

$$L(\kappa_1) = 1 - \frac{\Pi(\hat{r}, \hat{\bar{r}})}{\Pi(\hat{r}_1, \hat{r}_2)}.$$  \hfill (B15)

Note that both $\Pi(\hat{r}, \hat{\bar{r}})$ and $\Pi(\hat{r}_1, \hat{r}_2)$ are function of $\kappa_1$.

**Proposition B3 (Upper bound on physician revenue gap).** Consider a heterogeneous patient panel with $q_i \in [q^- (c, \Delta), q^+ (c, \Delta)], i = 1, 2$ and $q_1 \leq q^\ast < q_2$ in a proportional compensation setting. Then, for $T_i \geq \frac{1}{2} T_h$, the relative loss of revenue for the physician resulting from applying the same RVI to both patient groups, $L(\kappa_1)$, has the following properties:

a) For $\kappa_1 \leq \hat{\kappa}_1$, the relative loss of revenue is

$$L(\kappa_1) = 1 - \frac{\bar{R}}{1 + (\bar{R} - 1) \left( \frac{1}{1 + \kappa_1 (\xi(q_1) - 1)} \right)}.$$  \hfill (B16)
an increasing function of $\kappa_1$.

b) For $\kappa_1 > \hat{\kappa}_1$, the relative loss of revenue is

$$L(\kappa_1) = 1 - \frac{1}{1 + (\bar{R} - 1) \left( \frac{1}{\xi(q_2) + \kappa_1(\bar{\xi}(q_1) - \xi(q_2))} \right)},$$  \hspace{1cm} (B17)

a decreasing function of $\kappa_1$.

c) Therefore, the relative loss is maximized at $\kappa_1 = \hat{\kappa}_1$ and is given by

$$\epsilon^{U} = L(\hat{\kappa}_1) = \frac{\hat{\kappa}_1 (\bar{R} - 1) (1 - \xi(q_1))}{\bar{R} + \hat{\kappa}_1 (\xi(q_1) - 1)}. \hspace{1cm} (B18)$$

Parts a) and b) of Proposition B3 provide analytical expressions for the physician revenue loss resulting from applying a single RVI value to a heterogeneous patient panel: one for when the majority of the patients on the panel are “healthy” ($\kappa_1 > \hat{\kappa}_1$) and the other one for when the majority of patients are “sick” ($\kappa_1 \leq \hat{\kappa}_1$). They also show that the loss of revenue is increasing in $\kappa_1$ for $\kappa_1 \leq \hat{\kappa}_1$ and decreasing in $\kappa_1$ for $\kappa_1 > \hat{\kappa}_1$.

Given these results, part c) of Proposition B3 provides an upper bound on the fraction of physician revenue that is lost when the “uniform RVI” policy is applied in a setting with sufficiently heterogeneous patient population. Figure B5a shows the changes in this upper bound as a function of group 1’s health indicator, $q_1$. As the value of $q_1$ increases, the heterogeneity between the two patient groups decreases and so does the value of customizing RVIs for different patient groups. Note also that the “uniform RVI” policy produces no loss of revenue when $q_1 = \bar{q}$, because at that point the physician becomes indifferent to setting the RVI at $T_l$ or at $T_h$ for patients in group 1 (Corollary B1). Therefore, $T_l$ is the optimal RVI for all patients under both RVI policies.

Figure B5b plots the upper bound as a function of $\delta$, the proportion of the fee-for-service component in physician compensation. This figure suggests that RVI customization has the most pronounced impact on physician revenue when capitation payments dominate the compensation structure. In contrast, if the physician is compensated exclusively on a fee-for-service basis, there is no gap in revenue between the uniform and customized RVI policies, since, under the proportional fee-for-service structure, the compensation rate is the same for both urgent and routine office visits.

Note that $\epsilon^{U}$ is a tight bound only when $\kappa_1 = \hat{\kappa}_1$. For instance, $\epsilon^{U}$ is a conservative bound when $\kappa_1$ equals 0 or 1, because there is no revenue gap for these $\kappa_1$ values. Therefore, $\epsilon^{U}$ is a useful bound for cases in which the exact value of $\kappa_1$ while being difficult to establish with certainty, is unlikely to be close to 0 or 1.

As we examine how RVI customization affects system outcomes, it is important to discuss the results in terms of welfare losses. In Section 5.2, we defined patient welfare, a concept that is tied to changes in both panel health and panel size. Figure B4 shows that panel customization can indeed be welfare reducing for patients. In particular, for $\kappa_1 > \hat{\kappa}_1$, panel size decreases, and there will be patients that the physician will not be able to cover with RVI customization that would have otherwise been included in the physician’s panel. If the cost of not being included in the physician’s panel, $\nu$, is sufficiently large, then patient welfare is reduced under RVI customization. Physician welfare in our model is characterized by revenue, so RVI customization is welfare increasing for the physician. In addition, we showed in Proposition B3 and Figure B5 that the higher the capitation portion of physician revenue, the larger the bound on the physician revenue loss from using a uniform RVI policy instead of RVI customization.
B.3. E-Visits for Heterogeneous Patient Panel with Two Flexible Patient Groups

In the following analysis, we focus on characterizing the impact of e-visits in a heterogeneous setting with two patient groups that are both flexible in the absence of e-visits. In particular, we will look at three separate settings: the setting where the influence of e-visits is moderate enough to preserve patient flexibility in both groups, the setting where e-visits remove the flexibility of one of the patient groups, and, finally, the setting where both patient groups become inflexible after e-visits. Note that the results that follow can be extended to include settings in which at least one of the patient groups is inflexible before e-visits are introduced. Some of these cases are trivial, e.g., a patient group that is inflexible with $T_l$ before e-visits is guaranteed to stay inflexible with $T_l$ under e-visits, and thus the physician has no choice for the RVI of this patient group. Also, as we showed in Proposition 3, it is possible that a patient group that is inflexible in the absence of e-visits becomes flexible after e-visits are introduced. Our results on the optimal RVI values under e-visits are also applicable in such a setting, but we exclude them for brevity.

Note that as we showed in Proposition 2 the patients’ choice of e-visit adoption is a function of patient health level $q$. Therefore, it is possible that one group of patients chooses to adopt e-visits while the other one does not adopt e-visits. In the next proposition, we focus on the case where both patient groups choose to adopt e-visits.

For simplicity, we assume that the service times and the physician compensation amounts associated with e-visits are the same for both patient groups.

**Proposition B4 (Heterogeneous patient panel with e-visits).** Consider a setting where patients choose to adopt e-visits, and the patient panel is heterogeneous with $q_i \in [q^-(c, \Delta), q^+(c, \Delta)]$, $i = 1, 2$, $q_1 < q_2$, and (29) holds.

a) Suppose that $\alpha_i^x \leq \bar{\alpha}_i^x(q_2)$. Then, the optimal RVI values in (38)-(40) are given by

$$\hat{\tau}_i^x = \begin{cases} T_h, & q_i \leq \frac{q^+}{(1+\Sigma_\alpha)^+}, \\ T_l, & \text{otherwise}, \end{cases}$$

(B19)
\[
\tilde{r}_2 = \begin{cases} 
T_h, & q_1 \leq \frac{\theta^T - q_1 \left(1+\sum e\right)}{(1+\sum e) \theta^T - \kappa_1 \sum e - q_1 (1+\sum e)} , \\
T_l, & \text{otherwise}, 
\end{cases}
\]  
(B20)

where \(x^+ = \max(x, 0)\).

b) Suppose that \(\tilde{\alpha}^e_\tau(q_2) < \alpha_\tau^e \leq \tilde{\alpha}^e_\tau(q_1)\). Then the optimal RVI values in (38)-(40) are given by

\[
\tilde{r}_1 = \begin{cases} 
T_h, & q_1 \leq \frac{\theta^T - q_1 \left(1+\sum e\right)}{(1+\sum e) \theta^T - \kappa_1 \sum e - q_1 (1+\sum e)} , \\
T_l, & \text{otherwise}, 
\end{cases}
\]  
(B21)

and \(\tilde{r}_2 = T_l\).

c) Suppose that \(\alpha_\tau^e > \tilde{\alpha}^e_\tau(q_1)\). Then the optimal RVI values in (38)-(40) are given by \(\tilde{r}_1 = \tilde{r}_2 = T_l\).

Proposition B4 describes the physician’s choice of optimal RVI values under e-visits. In particular, there are three parts in this proposition: part a) describes a setting in which e-visits replace a relatively small fraction of routine office visits (\(\alpha_\tau^e \leq \tilde{\alpha}^e_\tau(q_2)\)). In such a setting, patients remain flexible with respect to RVI values under e-visits, and the structure of the optimal RVI values stays similar to the one described in Proposition B2. As the fraction of routine office visits replaced by e-visits increases, some patient groups may become inflexible under e-visits, with an RVI of \(T_l\) as discussed in Proposition 3. Parts b) and c) of Proposition B4 describe the physician’s optimal choice of RVI when at least one of the patient groups is inflexible, demanding frequent office visits with RVI equal to \(T_l\).

The results of Proposition B4 can be simplified in the case where “proportional” compensation rates in the absence of e-visits are augmented by the proportional compensation for e-visits (i.e., \(R^r = R_u = R^r_{re}\) and \(R^d = 0\)).

**Corollary B2.** Consider a heterogeneous patient panel with \(q_i \in [q^- (c, \Delta), q^+ (c, \Delta)], i = 1, 2, q_1 < q_2\), and (29) holds in the presence of e-visits under “proportional” compensation (\(R^r = R_u = R^r_{re}\) and \(R^d = 0\)). The optimal RVI values in (38)-(40) are given by

\[
\tilde{r}_1 = \begin{cases} 
T_h, & q_1 < \tilde{q}_1^e, \alpha_\tau^e \leq \tilde{\alpha}^e_\tau(q_1), \\
T_l, & \text{otherwise}, 
\end{cases}
\]  
(B22)

and

\[
\tilde{r}_2 = \begin{cases} 
T_h, & q_2 < \tilde{q}_2^e, \alpha_\tau^e \leq \tilde{\alpha}^e_\tau(q_2), \\
T_l, & \text{otherwise}. 
\end{cases}
\]  
(B23)

Similar to Corollary B1, Corollary B2 shows that under “proportional” compensation for all types of visit, the choice of optimal RVI for the two patient groups is decoupled: the optimal RVI values for each patient group depend only on that group’s characteristics. Note that under proportional e-visit compensation patients are guaranteed to adopt e-visits because \(R^e < R^r\) and \(R^d = 0\).
Proof of Proposition B1

When a physician applies the same RVI value \( r \) to both patient groups she can choose either \( r = r_1 = r_2 = T_h \) or \( r = r_1 = r_2 = T_i \). Therefore, we need to compare the values of the objective function (13) for \( r = r_1 = r_2 = T_i \) and \( r = r_1 = r_2 = T_h \). For \( r = r_1 = r_2 = T_i \), the objective function value is

\[
(1 - \delta) R^d + \delta \frac{R^c}{T_i},
\]

while for \( r = r_1 = r_2 = T_h \), the objective function value is

\[
(1 - \delta) R^d + \delta \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right) + \frac{\delta R^c}{T_i},
\]

for \( q_i \leq \frac{q}{(1 + \Sigma)^r}, \ i = 1, 2 \), then

\[
(1 - \delta) R^d + \delta \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right) \geq \frac{(1 - \delta) R^d + \delta \frac{R^c}{T_i}}{\frac{r}{T_i}},
\]

which is equivalent to

\[
(1 - \delta) R^d + \delta \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right) \geq \left( \frac{(1 - \delta) R^d + \delta \frac{R^c}{T_i}}{\frac{r}{T_i}} \right) \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right).
\]

For \( r = r_1 = r_2 = T_h \) to be optimal, we need to have, from (B24) and (B25),

\[
(1 - \delta) R^d + \delta \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right) + \frac{\delta R^c}{T_i} > \frac{(1 - \delta) R^d + \delta \frac{R^c}{T_i}}{\frac{r}{T_i}},
\]

or equivalently

\[
(1 - \delta) R^d + \delta \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right) + \frac{\delta R^c}{T_i} > \frac{(1 - \delta) R^d + \delta \frac{R^c}{T_i}}{\frac{r}{T_i}} \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right),
\]

which follows from (B27).

The optimality of \( r = r_1 = r_2 = T_i \) for the setting \( \frac{q_1}{(1 + \Sigma)^r} < q_1 < q_2 \) is established in the same fashion.

For \( q_1 \leq \frac{q}{(1 + \Sigma)^r} < q_2 \), we define the following notation:

\[
x_i = \delta \left( \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i} \right),
\]

\[
y_i = \frac{(1 - q_1)r^r + q_1 R^u}{(1 - q_1)T_h + q_1 T_i},
\]

\[
b = \frac{(1 - \delta) R^d + \delta \frac{R^c}{T_i}}{\frac{r}{T_i}},
\]

\[
a = (1 - \delta) R^d.
\]

Then, we simplify (B28) to

\[
\frac{a + \kappa_1 x_1 + \kappa_2 x_2}{\kappa_1 y_1 + \kappa_2 y_2} > b,
\]

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which simplifies to
\[ a + \kappa_1 x_1 + \kappa_2 x_2 > b (\kappa_1 y_1 + \kappa_2 y_2). \]  
(B36)

We re-express (B36) as
\[ \kappa_1 (x_1 - by_1) + \kappa_2 (x_2 - by_2) > -a, \]  
(B37)

which, given that \( \kappa_2 = 1 - \kappa_1 \), is equivalent to
\[ \kappa_1 ((x_1 - by_1) - (x_2 - by_2)) > -a - (x_2 - by_2). \]  
(B38)

Note that because of the conditions on \( q_1 \) and \( q_2 \) in this setting, from (B27) we have \( a + x_1 - by_1 \geq 0 \) and \( a + x_2 - by_2 < 0 \). Therefore, \( (x_1 - by_1) - (x_2 - by_2) > 0 \). We further simplify (B38) to
\[ \kappa_1 > \frac{1}{1 - \frac{a + (x_1 - by_1)}{a + (x_2 - by_2)}}. \]  
(B39)

The term in (B39) is \( \hat{k_1} \), and we simplify it using (B46) and (B48) in the following way:
\[
\begin{align*}
  a + (x_i - by_i) & = \\
  (1 - \delta) R^d + \delta R^r {R^r \over T_i} \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) - \left( {\tau^r \over T_i} \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) \right) \left( (1 - \delta) R^d + \delta R^r \over T_i \right) = \\
  (1 - \delta) R^d + \delta R^r {R^r \over T_i} \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) - \left( {1 \over T_i} \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) \right) \left( (1 - \delta) R^d T_i + \delta R^r \right) = \\
  (1 - \delta) R^d T_i \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) - \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) \left( (1 - \delta) R^d \over \delta R^r \right) + 1 = \\
  (R - 1) + \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) - \left( 1 + {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) = \\
  \left( {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) - \left( {q_i - \bar{q}_i \over \bar{q}_i (Q^T - q_i)} \right) = \frac{q_i - \bar{q}_i}{Q^T - q_i} - 1 - \frac{q_i - \bar{q}_i}{Q^T - q_i} (R - 1) + (R - 1) = \\
  \left( {\bar{q}_i \over \bar{q}_i} \left( 1 - \frac{\bar{q}_i}{q_i} \right) - \frac{q_i}{ \bar{q}_i} \right) = \left( \frac{R - 1}{Q^T - q_i} \right) - \left( \frac{q_i (1 + \Sigma) - \bar{q}_i}{Q^T - q_i} \right). \tag{B40}
\end{align*}
\]

The result then follows from (B39).

**Proof of Proposition B2**

Note that since both (14) and (15) are increasing functions of \( N \), (14)-(16) is equivalent to
\[
\max_{r_1, r_2} \frac{(1 - \delta) R^d + \delta \kappa_1 \left( \rho^{r_1}_{1} (r_1) R^r + (1 - \rho^{r_1}_{1} (r_1)) R^u \right) \over \kappa_1 T_{1} (r_1)} + \delta \kappa_2 \left( \rho^{r_2}_{2} (r_2) R^r + (1 - \rho^{r_2}_{2} (r_2)) R^u \right) \over \kappa_2 T_{2} (r_2)}
\]  
(B41)

s.t. \( r_1, r_2 \in \{ T_i, T_h \} \),
\]  
(B42)

with the optimal panel size determined by
\[
\hat{N} = \frac{A}{\kappa_1 \left( \rho^{r_1}_{1} (r_1) R^r + (1 - \rho^{r_1}_{1} (r_1)) R^u \right) \over \kappa_1 T_{1} (r_1)} + \kappa_2 \left( \rho^{r_2}_{2} (r_2) R^r + (1 - \rho^{r_2}_{2} (r_2)) R^u \right) \over \kappa_2 T_{2} (r_2)}.
\]  
(B43)
Let $S_l (S_h)$ be the set of patient group indices for which the optimal revisit interval is set at $T_l (T_h)$. Since

$$\rho_i (T_l) = 1, T_i (T_l) = T_l, i = 1, 2,$$  \hspace{1cm} \text{(B44)}

and

$$\rho_i (T_h) = 1 - q_i, T_i (T_h) = q_i T_l + (1 - q_i) T_h, i = 1, 2,$$  \hspace{1cm} \text{(B45)}

the expression in the numerator of objective function in (B41) can be expressed as

$$\frac{\rho_i (r_i) R^r + (1 - \rho_i (r_i)) R^u}{T_i (r_i)} = \frac{R^r}{T_i} \left( 1 + \frac{q_i - \bar{q}^R}{\bar{q}^R (Q^R - q_i)} Y_i \right),$$  \hspace{1cm} \text{(B46)}

where

$$Y_i = \begin{cases} 0, & i \in S_l, \\ 1, & i \in S_h. \end{cases}$$  \hspace{1cm} \text{(B47)}

We can re-write the expression in the denominator of (B41) as

$$\frac{1}{T_i (r_i)} (\rho_i (r_i) \tau^r + (1 - \rho_i (r_i)) \tau^u) = \frac{\tau^r}{T_i} \left( 1 + \frac{q_i - \bar{q}^R}{\bar{q}^R (Q^R - q_i)} Y_i \right).$$  \hspace{1cm} \text{(B48)}

Introducing

$$R_i = \kappa_i \left( \frac{q_i - \bar{q}^R}{\bar{q}^R (Q^R - q_i)} \right), i = 1, 2,$$  \hspace{1cm} \text{(B49)}

$$A_i = \kappa_i \left( \frac{q_i - \bar{q}^R}{\bar{q}^R (Q^R - q_i)} \right), i = 1, 2,$$  \hspace{1cm} \text{(B50)}

we note that the optimization problem (B41)-(B42) is equivalent to

$$\max_{Y_1, Y_2} \frac{\bar{R} + R_1 Y_1 + R_2 Y_2}{1 + A_1 Y_1 + A_2 Y_2}$$  \hspace{1cm} \text{(B51)}

s.t. $Y_1, Y_2 \in \{0, 1\}.$  \hspace{1cm} \text{(B52)}

Note that the optimal combination of $Y_1$ and $Y_2$ values is one of the four possible ones: $(0, 0), (1, 0), (0, 1)$ and $(1, 1)$. The four values for the objective function corresponding to those combinations are $\bar{R}, \frac{R_1}{1 + A_1}, \frac{R_2}{1 + A_2},$ and $\frac{R_1 + R_2}{1 + A_1 + A_2}$. In order to determine which of these four values is the highest, we will look at several different cases describing the signs of $R_1, A_1, R_2,$ and $A_2$. Note that the signs of these quantities are determined by the values of $q_i - \bar{q}^r$ and $q_i - \bar{q}^R, i = 1, 2.$ For the analysis below, we will need the following result.

**Lemma B1.** For $q_i \leq \bar{q}^R, i = 1, 2,$ we have

$$|R_1| + |R_2| \leq \bar{R},$$  \hspace{1cm} \text{(B53)}

and, for $q_i \leq \bar{q}^r, i = 1, 2,$ we have

$$|A_1| + |A_2| \leq 1.$$  \hspace{1cm} \text{(B54)}

Proof of Lemma B1:

Note that for $q_i \leq \bar{q}^R,$

$$|R_i| = \kappa_i \frac{\bar{q}^R - q_i}{\bar{q}^R (Q^R - q_i)}.$$

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is an decreasing function of \( q_i \) (due to \( Q^T > 1 > q^R \)). Then, we have
\[
|\mathcal{R}_i| = \kappa_i \left(\frac{q^R - q_i}{q^R (Q^T - q_i)}\right) \leq \frac{\kappa_i}{Q^T}.
\]
(B56)

Thus,
\[
\bar{R} - |\mathcal{R}_1| - |\mathcal{R}_2| \geq 1 - \frac{\kappa_1}{Q^T} - \frac{\kappa_2}{Q^T} \geq \left(1 - \frac{1}{Q^T}\right) > 0.
\]
(B57)

In a similar fashion, for \( q_i \leq \bar{q}^r \), we have
\[
|\mathcal{A}_i| = \kappa_i \left(\frac{\bar{q}^r - q_i}{\bar{q}^r (Q^T - q_i)}\right) \leq \frac{\kappa_i}{Q^T},
\]
(B58)

and
\[
\bar{I} - |\mathcal{A}_1| - |\mathcal{A}_2| \geq 1 - \frac{\kappa_1}{Q^T} - \frac{\kappa_2}{Q^T} \geq \left(1 - \frac{1}{Q^T}\right) > 0.
\]
(B59)

Assuming that \( q_1 < q_2 \), we will analyze 12 separate cases. Note that \( \Sigma \) defined in (21) is non-negative if and only if \( \bar{q}^r \leq q^R \).

Case 1: \( \bar{q}^r \leq \bar{q}^R \), \( q_1 < q_2 < \bar{q}^r \).

In this case, we have \( \mathcal{R}_i, \mathcal{A}_i < 0 \), \( i = 1, 2 \). Then,
\[
\frac{\bar{R} + \mathcal{R}_1}{1 + \mathcal{A}_1} = \frac{\bar{R} - |\mathcal{R}_1|}{1 - |\mathcal{A}_1|}, \quad (B60)
\]
\[
\frac{\bar{R} + \mathcal{R}_2}{1 + \mathcal{A}_2} = \frac{\bar{R} - |\mathcal{R}_2|}{1 - |\mathcal{A}_2|}, \quad (B61)
\]
\[
\frac{\bar{R} + \mathcal{R}_1 + \mathcal{R}_2}{1 + \mathcal{A}_1 + \mathcal{A}_2} = \frac{\bar{R} - |\mathcal{R}_1| - |\mathcal{R}_2|}{1 - |\mathcal{A}_1| - |\mathcal{A}_2|}, \quad (B62)
\]

where
\[
|\mathcal{R}_i| = \kappa_i \left(\frac{1 - \frac{q_i}{q^R}}{Q^T - q_i}\right), \quad i = 1, 2, \quad (B63)
\]
\[
|\mathcal{A}_i| = \kappa_i \left(\frac{1 - \frac{\bar{q}^r}{q^R}}{Q^T - q_i}\right), \quad i = 1, 2, \quad (B64)
\]

Note that because \( q_1 < q_2 \) and \( \bar{q}^r \leq q^R \), we have
\[
|\mathcal{R}_i| \geq |\mathcal{A}_i|, \quad i = 1, 2, \quad (B65)
\]
\[
\frac{|\mathcal{R}_2|}{|\mathcal{A}_2|} > \frac{|\mathcal{R}_1|}{|\mathcal{A}_1|} \geq 1. \quad (B66)
\]

Suppose that \( \bar{R} \geq \frac{|\mathcal{R}_2|}{|\mathcal{A}_2|} \). Then,
\[
\bar{R} \leq \frac{\bar{R} - |\mathcal{R}_1|}{1 - |\mathcal{A}_1|}, \quad (B67)
\]
\[
\bar{R} \leq \frac{\bar{R} - |\mathcal{R}_2|}{1 - |\mathcal{A}_2|}. \quad (B68)
\]

Note that, in this case,
\[
\frac{\bar{R} - |\mathcal{R}_1| - |\mathcal{R}_2|}{1 - |\mathcal{A}_1| - |\mathcal{A}_2|} - \frac{\bar{R} - |\mathcal{R}_1|}{1 - |\mathcal{A}_1|} = \frac{|\mathcal{A}_2| \left(\bar{R} - |\mathcal{R}_1|\right) - |\mathcal{R}_2| \left(1 - |\mathcal{A}_1|\right)}{(1 - |\mathcal{A}_1|) \left(1 - |\mathcal{A}_1| - |\mathcal{A}_2|\right)}
\]
\begin{align*}
|A_2| \left( \frac{r_2}{|A_2|} - |R_1| \right) - |R_2| (1 - |A_1|) & \leq |A_1||A_2| \left( \frac{r_2}{|A_2|} - \frac{r_1}{|A_1|} \right) = \frac{|A_1||A_2| (r_2 - r_1)}{(1 - |A_1|) (1 - |A_1| - |A_2|)} > 0, \\
\text{and} \\
\frac{\bar{R} - |R_1| - |R_2|}{1 - |A_1| - |A_2|} - \frac{\bar{R} - |R_2|}{1 - |A_2|} & = \frac{|A_1| (\bar{R} - |R_2|) - |R_1| (1 - |A_2|)}{(1 - |A_1|) (1 - |A_1| - |A_2|)} \\
& = \frac{|A_1| (\bar{R} - |R_2| - \frac{r_1}{|A_1|})}{(1 - |A_1| - |A_2|)} > 0. 
\end{align*}

Thus, the optimal combination of \( Y_1 \) and \( Y_2 \) is \((1,1)\), corresponding to the optimal RVI values \( \hat{r}_1 = \hat{r}_2 = T_h \).

Now, suppose that \( \frac{r_1}{|A_1|} \leq \frac{r_2}{|A_2|} \). Note that

\begin{equation}
\frac{1 - \frac{q_1}{\bar{q}^\tau}}{1 - \frac{q_2}{\bar{q}^\tau}} \leq \bar{R}
\end{equation}

is equivalent to

\begin{equation}
q_1 \leq \bar{q}^\tau \frac{\bar{R} - 1}{\bar{R} - \frac{\bar{q}^\tau}{1 + \Sigma}} = \frac{\bar{q}^\tau}{1 + \Sigma},
\end{equation}

with \( \Sigma \) defined by (21). In a similar fashion,

\begin{equation}
\frac{1 - \frac{q_2}{\bar{q}^\tau}}{1 - \frac{q_2}{\bar{q}^\tau}} > \bar{R}
\end{equation}

is equivalent to

\begin{equation}
q_2 > \frac{\bar{q}^\tau}{1 + \Sigma}.
\end{equation}

Then,

\begin{align*}
\bar{R} & \leq \frac{\bar{R} - |R_1|}{1 - |A_1|}, \\
\bar{R} & > \frac{\bar{R} - |R_2|}{1 - |A_2|},
\end{align*}

and

\begin{align*}
\frac{\bar{R} - |R_1| - |R_2|}{1 - |A_1| - |A_2|} - \frac{\bar{R} - |R_1|}{1 - |A_1|} & = \frac{|A_2| (\bar{R} - |R_1|) - |R_2| (1 - |A_1|)}{(1 - |A_1| - |A_2|)} \\
& = \frac{|A_2| (1 - |A_1|) \left( \frac{\bar{R} - |R_1|}{1 - |A_1|} - \frac{r_2}{|A_2|} \right)}{(1 - |A_2|) (1 - |A_1| - |A_2|)}. 
\end{align*}

We define

\begin{equation}
\bar{R}^* = \frac{\bar{R} + |R_1|}{1 + |A_1|} = \frac{\bar{R} - \frac{r_1}{\bar{q}^{\tau} - q_1} \left( 1 - \frac{q_2}{\bar{q}^\tau} \right)}{1 - \frac{r_1}{\bar{q}^{\tau} - q_1} \left( 1 - \frac{q_2}{\bar{q}^\tau} \right)},
\end{equation}

and observe that, in the setting with \( R_1, A_1 < 0 \) we have

\begin{equation}
\bar{R}^* = \frac{\bar{R} - |R_1|}{1 - |A_1|}.
\end{equation}
For $\left| \frac{\pi_1}{A_1} \right| \leq \tilde{R}$,

$$\tilde{R}^* - \tilde{R} = \frac{\tilde{R} - \left| \frac{\pi_1}{A_1} \right|}{1 - \left| \frac{\pi_1}{A_1} \right|} - \tilde{R} = \frac{|A_1|}{1 - \left| \frac{\pi_1}{A_1} \right|} \left( \tilde{R} - \left| \frac{\pi_1}{A_1} \right| \right) > 0,$$

(B80)

we see that the optimal combination of $Y_1$ and $Y_2$ is $(1, 0)$ for $\tilde{R} \leq \tilde{R}^* < \left| \frac{\pi_2}{A_2} \right|$ and $(1, 1)$ for $\tilde{R} < \left| \frac{\pi_2}{A_2} \right| \leq \tilde{R}^*$, so that the optimal RVI values $\hat{r}_1 = T_h$ and $\hat{r}_2 = T_1$ for $\tilde{R} \leq \tilde{R}^* < \left| \frac{\pi_2}{A_2} \right|$ and $\hat{r}_1 = \hat{r}_2 = T_h$ for $\tilde{R} < \left| \frac{\pi_2}{A_2} \right| \leq \tilde{R}^*$.

Note that

$$\left| \frac{\pi_2}{A_2} \right| < \frac{\tilde{R} - \left| \frac{\pi_1}{A_1} \right|}{1 - \left| \frac{\pi_1}{A_1} \right|} = \tilde{R}^*, \tag{B81}$$

is equivalent to

$$q_2 < \frac{q^r}{\bar{R}^* - \frac{q^r}{\bar{q}^r}}. \tag{B82}$$

The expression on the right-hand side of (B82) can be re-written as

$$\frac{\tilde{R}^* - 1}{\bar{R}^* - \frac{q^r}{\bar{q}^r}} = \tilde{R} - 1 + \left( \tilde{R} - 1 - \frac{\tilde{R} - 1}{\bar{R}^* - \frac{q^r}{\bar{q}^r}} \right). \tag{B83}$$

Further,

$$\tilde{R}^* - 1 = \tilde{R} - \left| \frac{\pi_1}{A_1} \right| - 1 = \frac{\tilde{R} - 1 - \left| \frac{\pi_1}{A_1} \right| + |A_1|}{1 - \left| \frac{\pi_1}{A_1} \right|} > 0, \tag{B84}$$

and

$$\tilde{R}^* - \frac{\bar{q}^r}{\bar{q}^r} = \tilde{R} - \left| \frac{\pi_1}{A_1} \right| - \frac{\bar{q}^r}{\bar{q}^r} = \tilde{R} - \frac{\bar{q}^r}{\bar{q}^r} - \frac{\bar{q}^r}{\bar{q}^r} + \frac{\bar{q}^r}{\bar{q}^r} |A_1| > 0, \tag{B85}$$

since $\tilde{R}^* \geq \tilde{R} \geq 1$, so that

$$\frac{\tilde{R} - 1}{\bar{R} - \frac{q^r}{\bar{q}^r}} = \frac{\tilde{R} - 1 - \left| \frac{\pi_1}{A_1} \right| + |A_1|}{\bar{R} - \frac{q^r}{\bar{q}^r} - \left| \frac{\pi_1}{A_1} \right| + \frac{q^r}{\bar{q}^r} |A_1|} = \frac{\tilde{R} - 1 - \frac{q^r}{\bar{q}^r} \left( \frac{\pi_1}{A_1} - q_1 \right) \left( 1 - \frac{q^r}{\bar{q}^r} \right)}{\bar{R} - 1 - \frac{q^r}{\bar{q}^r} - \left( \frac{\pi_1}{A_1} - q_1 \right) \left( 1 - \frac{q^r}{\bar{q}^r} \right)}$$

$$= 1 - \frac{\bar{q}^r}{\bar{q}^r} \left( \frac{\pi_1}{A_1} - q_1 \right) \Sigma = \frac{Q_T - q_1 - \frac{q^r}{\bar{q}^r} \kappa_1 \Sigma}{(1 + \Sigma) (Q^T - q_1) - \kappa_1 \Sigma} = \frac{Q_T - q_1 \left( 1 + \frac{q^r}{\bar{q}^r} \Sigma \right)}{(1 + \Sigma) Q^T - \kappa_1 \Sigma - q_1 \left( 1 + \Sigma \right)}. \tag{B86}$$

Note that for $\Sigma > 0$ and $q_1 \leq \frac{q^r}{1+\Sigma}$, the denominator of the expression on the right-hand side of (B86) is positive. Thus, (B82) is equivalent to (B11) for $\Sigma > 0$. Note that the right-hand side of (B86) is a decreasing function of $q_1$, and it reaches its maximum value on the interval $q_1 \in [0, \bar{q}^r]$ at $q_1 = 0$. That maximum value,

$$\left( 1 + \Sigma \right) Q^T - \kappa_1 \Sigma \tag{B87}$$

in decreasing in $Q^T$ and increasing in $\kappa_1$, and, thus, reaches its highest value, equal to 1, at $Q^T = \kappa_1 = 1$. Thus

$$q_2 \leq \frac{q^r}{\bar{q}^r} \left( \frac{Q_T - q_1 \left( 1 + \frac{q^r}{\bar{q}^r} \Sigma \right)}{(1 + \Sigma) Q^T - \kappa_1 \Sigma - q_1 \left( 1 + \Sigma \right)} \right) \tag{B88}$$
implies that \( q_2 < \tilde{q}^r \).

Finally, suppose that \( 1 \leq \tilde{R} \leq \frac{|R_1|}{|A_1|} \). Then,

\[
\tilde{R} > \frac{\tilde{R} - |R_1|}{1 - |A_1|}, \quad \text{(B89)}
\]

\[
\tilde{R} > \frac{\tilde{R} - |R_2|}{1 - |A_2|}, \quad \text{(B90)}
\]

and

\[
\frac{\tilde{R} - |R_1| - |R_2|}{1 - |A_1| - |A_2|} - \tilde{R} = \frac{-|R_1| - |R_2| + \tilde{R}|A_1| + \tilde{R}|A_2|}{1 - |A_1| - |A_2|} < 0,
\]

so that the optimal combination of \( Y_1 \) and \( Y_2 \) is \( (0,0) \), and the optimal RVI values are \( \hat{\tau}_1 = \hat{\tau}_2 = T_l \).

Case 2: \( \tilde{q}^r < \tilde{q}^R, q_1 < \tilde{q}^r \leq q_2 \leq \tilde{q}^R \).

In this case, we have \( R_1, R_2, A_1 < 0 \), and \( A_2 \geq 0 \). Then,

\[
\tilde{R} + \frac{R_1}{1 + A_1} = \frac{\tilde{R} - |R_1|}{1 - |A_1|}, \quad \text{(B92)}
\]

\[
\tilde{R} + \frac{R_2}{1 + A_2} = \frac{\tilde{R} - |R_2|}{1 - |A_2|}, \quad \text{(B93)}
\]

\[
\tilde{R} + \frac{R_1 + R_2}{1 + A_1 + A_2} = \frac{\tilde{R} - |R_1| - |R_2|}{1 - |A_1| + |A_2|}, \quad \text{(B94)}
\]

where

\[
|R_1| = \kappa_i \left( 1 - \frac{q_i}{Q^T - q_i} \right), \quad i = 1, 2,
\]

\[
|A_1| = \kappa_1 \left( 1 - \frac{q_1}{Q^T - q_1} \right),
\]

\[
|A_2| = \kappa_2 \left( \frac{q_2}{Q^T - q_2} - 1 \right).
\]

Note that the optimal value for \( Y_2 \) is 0, and the optimal value of \( Y_1 \) depends on the relative values of \( \tilde{R} \) and \( \frac{|R_1|}{1 - |A_1|} \). We have

\[
\tilde{R} + \frac{R_1}{1 + A_1} - \tilde{R} = \frac{\tilde{R}|A_1| - |R_1|}{1 - |A_1|}.
\]

Thus, the optimal combination of \( Y_1 \) and \( Y_2 \) is \( (1,0) \) for \( \tilde{R} \geq \frac{|R_1|}{|A_1|} \) and \( (0,0) \) for \( \tilde{R} < \frac{|R_1|}{|A_1|} \), so that the optimal RVI values \( \hat{\tau}_1 = \hat{\tau}_2 = T_l \) for \( \tilde{R} < \frac{|R_1|}{|A_1|} \) and \( \hat{\tau}_1 = T_h \) and \( \hat{\tau}_2 = T_l \) for \( \tilde{R} \geq \frac{|R_1|}{|A_1|} \). Note that, as we showed in (B72), \( \tilde{R} \geq \frac{|R_1|}{|A_1|} \) is equivalent to \( q_1 \leq \frac{q^r}{1 + \Sigma} \). Also, as we have shown earlier, \( q_2 > \tilde{q}^r \) ensures that

\[
q_2 > \tilde{q}^r \left( \frac{Q^T - q_1 - \frac{q_1^2}{4} \kappa_1 \Sigma}{(1 + \Sigma)(Q^T - q_1) - \kappa_1 \Sigma} \right) \equiv \frac{Q^T - q_1 + \frac{q_1^2}{4} \Sigma}{(1 + \Sigma) Q^T - \kappa_1 \Sigma - q_1 (1 + \Sigma)}.
\]

Case 3: \( \tilde{q}^r \leq \tilde{q}^R, q_1 < \tilde{q}^r \leq \tilde{q}^R \leq q_2 \).

In this case, we have \( R_1, A_1 < 0 \), and \( R_2, A_2 \geq 0 \). Then,

\[
\tilde{R} + \frac{R_1}{1 + A_1} = \frac{\tilde{R} - |R_1|}{1 - |A_1|}.
\]
Further, we have

\[
\frac{\bar{R} + \bar{R}_2}{1 + A_2} = \frac{\bar{R} + |\bar{R}_2|}{1 + A_2},
\]

(B101)

\[
\frac{\bar{R} + \bar{R}_1 + \bar{R}_2}{1 + A_1 + A_2} = \frac{\bar{R} - |\bar{R}_1| + |\bar{R}_2|}{1 - |A_1| + |A_2|},
\]

(B102)

where

\[
|\bar{R}_1| = \kappa_1 \left( 1 - \frac{q_1}{\bar{R} + q_1} \right),
\]

(B103)

\[
|\bar{R}_2| = \kappa_2 \left( \frac{q_1}{\bar{R} + q_1} - 1 \right),
\]

(B104)

\[
|A_1| = \kappa_1 \left( 1 - \frac{q_2}{\bar{R} + q_1} \right),
\]

(B105)

\[
|A_2| = \kappa_2 \left( \frac{q_2}{\bar{R} + q_1} - 1 \right).
\]

(B106)

Note that in this case we have

\[
\frac{|\bar{R}_2|}{|A_2|} \leq 1 \leq \frac{|\bar{R}_1|}{|A_1|}.
\]

(B107)

Suppose that \( \bar{R} \geq \frac{|\bar{R}_1|}{|A_1|} \). Then,

\[
\bar{R} \leq \frac{\bar{R} - |\bar{R}_1|}{1 - |A_1|},
\]

(B108)

\[
\bar{R} \geq \frac{\bar{R} + |\bar{R}_2|}{1 + |A_2|}.
\]

(B109)

Further, we have

\[
\frac{\bar{R} - |\bar{R}_1| + |\bar{R}_2|}{1 - |A_1| + |A_2|} - \frac{\bar{R} - |\bar{R}_1|}{1 - |A_1|} = \frac{-|A_2| (\bar{R} - |\bar{R}_1|) + |\bar{R}_2| (1 - |A_1|)}{1 - |A_1| - |A_2|} (1 - |A_1| - |A_2|) \leq 0,
\]

(B110)

Thus, the optimal combination of \( Y_1 \) and \( Y_2 \) is \((1, 0)\), corresponding to the optimal RVI values \( \hat{r}_1 = T_k, \hat{r}_2 = T_1 \).

Note that, as we showed in (B72), \( \bar{R} \geq \frac{|\bar{R}_1|}{|A_1|} \) is equivalent to \( q_1 \leq \frac{q_1^*}{1 + \kappa} \).

Now, suppose that \( \frac{|\bar{R}_2|}{|A_2|} \leq 1 \leq \frac{|\bar{R}_1|}{|A_1|} \). Then,

\[
\bar{R} \geq \frac{\bar{R} - |\bar{R}_1|}{1 - |A_1|},
\]

(B111)

\[
\bar{R} > \frac{\bar{R} + |\bar{R}_2|}{1 + |A_2|},
\]

(B112)

and

\[
\frac{\bar{R} - |\bar{R}_1| + |\bar{R}_2|}{1 - |A_1| + |A_2|} - \bar{R} = \frac{|A_1| (\bar{R} - \frac{|\bar{R}_1|}{|A_1|}) + |A_2| (\frac{|\bar{R}_2|}{|A_2|} - \bar{R})}{1 - |A_1| + |A_2|} \leq 0,
\]

(B113)

so that the optimal combination of \( Y_1 \) and \( Y_2 \) is \((0, 0)\), corresponding to the optimal RVI values \( \hat{r}_1 = T_1, \hat{r}_2 = T_1 \).

Case 4: \( q^* \leq \bar{q}^R, q^* \leq q_1 \leq q_2 \leq q^R \).
In this case, we have \( R_1, R_2 \leq 0 \), and \( A_1, A_2 \geq 0 \). Then,

\[
\frac{\bar{R} + R_1}{1 + A_1} = \frac{\bar{R} - |R_1|}{1 + |A_1|},
\]

\[
\frac{\bar{R} + R_2}{1 + A_2} = \frac{\bar{R} - |R_2|}{1 + |A_2|},
\]

\[
\frac{\bar{R} + R_1 + R_2}{1 + A_1 + A_2} = \frac{\bar{R} - |R_1| - |R_2|}{1 + |A_1| + |A_2|},
\]

and \( \bar{R} \) is the highest value among the four possibilities. Thus, optimal combination of \( Y_1 \) and \( Y_2 \) is \((0, 0)\), corresponding to the optimal RVI values \( \hat{r}_1 = T_1, \hat{r}_2 = T_i \).

Case 5: \( \bar{q}^r \leq \bar{q}^R, \bar{q}^r \leq q_1 \leq \bar{q}^R < q_2 \).

In this case, we have \( R_1 \leq 0, R_2 > 0 \), and \( A_1 \geq 0, A_2 > 0 \). Then,

\[
\frac{\bar{R} + R_1}{1 + A_1} = \frac{\bar{R} - |R_1|}{1 + |A_1|},
\]

\[
\frac{\bar{R} + R_2}{1 + A_2} = \frac{\bar{R} + |R_2|}{1 + |A_2|},
\]

\[
\frac{\bar{R} + R_1 + R_2}{1 + A_1 + A_2} = \frac{\bar{R} - |R_1| + |R_2|}{1 + |A_1| + |A_2|},
\]

and the optimal value of \( Y_1 \) is 0. The optimal value of \( Y_2 \) depends on the relative values of \( \bar{R} \) and \( \frac{\bar{R} + |R_2|}{1 + |A_2|} \).

Note that

\[
\frac{\bar{R} + |R_2|}{1 + |A_2|} - \bar{R} = \frac{|A_2| \left( \frac{|R_2|}{|A_2|} - \bar{R} \right)}{1 + |A_2|} < 0,
\]

since \( \bar{R} \geq 1 \), and, for \( \bar{q}^r \leq \bar{q}^R < q_2 \), we have \( \frac{|R_2|}{|A_2|} < 1 \). Thus, the optimal combination of \( Y_1 \) and \( Y_2 \) is \((0, 0)\), corresponding to the optimal RVI values \( \hat{r}_1 = T_1, \hat{r}_2 = T_i \).

Case 6: \( \bar{q}^r \leq \bar{q}^R, \bar{q}^R < q_1 < q_2 \).

In this case, we have \( R_1, R_2 > 0 \), and \( A_1, A_2 > 0 \). Then,

\[
|R_1| = \kappa_1 \left( \frac{q_1 - \bar{q}^R}{Q^T - q_1} \right),
\]

\[
|R_2| = \kappa_2 \left( \frac{q_2 - \bar{q}^R}{Q^T - q_2} \right),
\]

\[
|A_1| = \kappa_1 \left( \frac{q_1 - \bar{q}^R}{Q^T - q_1} \right),
\]

\[
|A_2| = \kappa_2 \left( \frac{q_2 - \bar{q}^R}{Q^T - q_1} \right),
\]

and

\[
\frac{|R_1|}{|A_1|} < \frac{|R_2|}{|A_2|} \leq 1 \leq \bar{R}.
\]

Note that in this case, we also have

\[
\frac{\bar{R} + R_1}{1 + A_1} = \frac{\bar{R} + |R_1|}{1 + |A_1|},
\]
\[
\frac{R + R_2}{1 + A_2} = \frac{\bar{R} + |R_2|}{1 + A_2}, \quad (B127)
\]
\[
\frac{R + R_1 + R_2}{1 + A_1 + A_2} = \frac{\bar{R} + |R_1| + |R_2|}{1 + |A_1| + |A_2|}. \quad (B128)
\]

Further, we have
\[
\frac{R + R_1}{1 + A_1} - \bar{R} = \frac{A_1 \left( \frac{\kappa_i}{Q^i} - \bar{R} \right)}{1 + A_1} \leq 0, \quad (B129)
\]
\[
\frac{R + R_2}{1 + A_2} - \bar{R} = \frac{A_2 \left( \frac{\kappa_2}{Q^2} - \bar{R} \right)}{1 + A_2} \leq 0, \quad (B130)
\]
\[
\frac{R + R_1 + R_2}{1 + A_1 + A_2} - \bar{R} = \frac{A_1 \left( \frac{\kappa_i}{Q^i} - \bar{R} \right) + A_2 \left( \frac{\kappa_2}{Q^2} - \bar{R} \right)}{1 + A_1 + A_2} \leq 0, \quad (B131)
\]

so that the optimal combination of \(Y_1\) and \(Y_2\) is \((0, 0)\), corresponding to the optimal RVI values \(\bar{r}_1 = T_i, \bar{r}_2 = T_i\).

**Case 7:** \(q^r > q^R\), \(q_1 < q_2 < q^R\).

In this case, we have \(R_i, A_i < 0, i = 1, 2\). Then,
\[
\frac{R + R_1}{1 + A_1} = \frac{\bar{R} - |R_1|}{1 - |A_1|}, \quad (B132)
\]
\[
\frac{R + R_2}{1 + A_2} = \frac{\bar{R} - |R_2|}{1 - |A_2|}, \quad (B133)
\]
\[
\frac{R + R_1 + R_2}{1 + A_1 + A_2} = \frac{\bar{R} - |R_1| - |R_2|}{1 - |A_1| - |A_2|}, \quad (B134)
\]

where
\[
|R_i| = \kappa_i \left( \frac{1 - \frac{q_i}{Q^i}}{q^r_q - q_i} \right), i = 1, 2, \quad (B135)
\]
\[
|A_i| = \kappa_i \left( \frac{1 - \frac{q_i}{Q^i}}{q^r_q - q_i} \right), i = 1, 2. \quad (B136)
\]

Note that because \(q_1 < q_2\) and \(q^r > q^R\), we have
\[
\frac{|R_1|}{|A_1|} < \frac{|R_2|}{|A_2|}, i = 1, 2, \quad (B137)
\]
\[
\frac{|R_2|}{|A_2|} < \frac{|R_1|}{|A_1|} < 1, \quad (B138)
\]

so that \(\bar{R} \geq 1 > \frac{|R_1|}{|A_1|} > \frac{|R_2|}{|A_2|}\). Then,
\[
\frac{\bar{R} - \bar{R} - |R_1|}{1 - |A_1|} \leq 0, \quad (B139)
\]
\[
\frac{\bar{R} - \bar{R} - |R_2|}{1 - |A_2|} \leq 0, \quad (B140)
\]
\[
\frac{\bar{R} - |R_1|}{1 - |A_1|} - \frac{\bar{R} - |R_1| - |R_2|}{1 - |A_1| - |A_2|} = \frac{|A_2| \left( \frac{\kappa_2}{A_2} - \frac{\bar{R} - |R_1|}{1 - |A_1|} \right)}{1 - |A_1| - |A_2|} \leq 0, \quad (B141)
\]
\[
\frac{\bar{R} - |R_2|}{1 - |A_2|} - \frac{\bar{R} - |R_1| - |R_2|}{1 - |A_1| - |A_2|} = \frac{|A_1| (\frac{Q}{|A_1|} - \frac{\bar{R} - |R_2|}{1 - |A_2|})}{1 - |A_1| - |A_2|}
\]
so that the optimal combination of \(Y_1 \) and \(Y_2\) is \((1,1)\), corresponding to the optimal RVI values \(\hat{r}_1 = T_h, \hat{r}_2 = T_h\).

Note that the conditions in this case also satisfy the conditions in (B11). \(\bar{q}^r > \bar{q}^\tau\) guarantees that \(\Sigma < 0\), so that \(q_i < \bar{q}^R < \bar{q}^r, i = 1,2\) will also guarantee \(q_i < (\frac{\bar{q}^r}{1 + \Sigma})\). Also, consider
\[
q_2 < \bar{q}^r \left(\frac{Q^r - q_1 \left(1 + \frac{\kappa_1}{\bar{q}^r} \Sigma\right)}{((1 + \Sigma) Q^r - \kappa_1 \Sigma - q_1 (1 + \Sigma))^{1+}}\right)
\]
in the case where
\[
(1 + \Sigma) Q^r - \kappa_1 \Sigma - q_1 (1 + \Sigma) > 0.
\]
First, suppose \(-1 < \Sigma < 0\). Then, right-hand side of (B143) is a decreasing function of \(q_1\), and the minimum of that expression on \(q_1 \in [0, \bar{q}^R]\) is reached at \(q_1 = \bar{q}^R\). This minimum is equal to
\[
\frac{Q^r - \bar{q}^R \left(1 + \frac{\kappa_1}{\bar{q}^r} \Sigma\right)}{((1 + \Sigma) Q^r - \kappa_1 \Sigma - \bar{q}^R (1 + \Sigma))^{1+}}.
\]
Then, since for \(\Sigma < 0\), (B145) is a decreasing function of \(\kappa_1\), we have its smallest value for \(\kappa_1 = 1\):
\[
\frac{Q^r - \bar{q}^R \left(1 + \frac{1}{\bar{q}^r} \Sigma\right)}{(1 + \Sigma) Q^r - \Sigma - \bar{q}^R (1 + \Sigma)}.
\]
Given that \(1 + \Sigma > 0\), and \(Q^r > 1\), we have \((1 + \Sigma) Q^r - \Sigma > 1 > \bar{q}^r\), so that (B146) is an increasing function of \(\bar{q}^R\). Taking the smallest possible value of \(\bar{q}^R = 0\), we get a lower bound on (B146):
\[
\frac{Q^r}{(1 + \Sigma) Q^r - \Sigma}.
\]
Since for \(\Sigma < 0\) (B147) is an increasing function of \(Q^r\), the smallest possible value of (B147) is obtained for \(Q^r = 1\) and is equal to 1. Thus, the right-hand side of (B143) is greater than or equal to \(\bar{q}^r\), and (B143) is satisfied for any \(q_2 < \bar{q}^R\).

Now, suppose that \(\Sigma \leq -1\). Then, we can re-express (B143) as
\[
q_2 < \bar{q}^r \left(\frac{Q^r - q_1 \left(1 - \frac{\kappa_1}{\bar{q}^r} |\Sigma|\right)}{(1 - |\Sigma|) Q^r + \kappa_1 |\Sigma| - q_1 (1 - |\Sigma|)}\right),
\]
As the right-hand side of (B148) is an increasing function of \(Q^r\), its lowest value is achieved for \(Q^r = 1\), and is given by
\[
\frac{1 - q_1 \left(1 - \frac{\kappa_1}{\bar{q}^r} |\Sigma|\right)}{(1 - |\Sigma|) + \kappa_1 |\Sigma| - q_1 (1 - |\Sigma|)}.
\]
The right-hand side of (B149) is a decreasing function of $\kappa_1$, and by setting $\kappa_1 = 1$, we get the lowest possible value
\[
\frac{1 - q_1 \left(1 - \frac{1}{\bar{q}^r} \Sigma\right)}{1 - q_1 \left(1 - \Sigma\right)},
\]
which is an increasing function of $q_1$, reaching its minimum, $\bar{q}^r$ at $q_1 = 0$. Thus, the right-hand side of (B143) is greater than or equal to $\bar{q}^r$.

**Case 8:** $\bar{q}^r > \bar{q}^R$, $q_1 < \bar{q}^R \leq q_2 < \bar{q}^r$.

In this case, we have $R_1, A_2, A_1 < 0$, and $R_2 \geq 0$. Then,
\[
\frac{\bar{R} + R_1}{1 + A_1} = \frac{\bar{R} - |R_1|}{1 - |A_1|},
\]
\[
\frac{\bar{R} + R_2}{1 + A_2} = \frac{\bar{R} + |R_2|}{1 - |A_2|},
\]
\[
\frac{\bar{R} + R_1 + R_2}{1 + A_1 + A_2} = \frac{\bar{R} - |R_1| + |R_2|}{1 - |A_1| - |A_2|},
\]
where
\[
|R_1| = \kappa_1 \left(1 - \frac{q_1}{\bar{q}^R} \right) Q^T - q_1, (B154)
\]
\[
|R_2| = \kappa_2 \left(\frac{q_2}{\bar{q}^R} - 1\right) Q^T - q_2, (B155)
\]
\[
|A_i| = \kappa_i \left(1 - \frac{q_i}{\bar{q}^R} \right) Q^T - q_i, i = 1, 2. (B156)
\]
Note that the optimal value for $Y_2$ is 1, and the optimal value of $Y_1$ depends on the relative values of $\frac{R + |R_2|}{1 - |A_2|}$ and $\frac{R - |R_2| + |R_2|}{1 - |A_2|}$. We have
\[
\frac{\bar{R} + |R_2|}{1 - |A_2|} - \frac{\bar{R} - |R_1| + |R_2|}{1 - |A_2|} = \frac{|A_1| \left(\frac{|R_1|}{A_1} - \frac{R + |R_2|}{1 - |A_2|}\right)}{1 - |A_1| - |A_2|} = \frac{|A_1| \left(\frac{|R_1|}{A_1} - \bar{R} + \bar{R} - \frac{R + |R_2|}{1 - |A_2|}\right)}{1 - |A_1| - |A_2|} \leq 0. (B157)
\]
Thus, the optimal combination of $Y_1$ and $Y_2$ is (1, 1), and the optimal RVI values $\bar{r}_1 = \bar{r}_2 = T_h$. Note that the conditions in this case also satisfy the conditions in (B11). $\bar{q}^r > \bar{q}^R$ guarantees that $\Sigma < 0$, so $q_1 < \bar{q}^r$ will also guarantee $q_1 < \frac{q^r}{(1 + \Sigma)^r}$, and, similarly to the analysis in Case 7, that
\[
q_2 < \bar{q}^r \left(\frac{Q^T - q_1 \left(1 + \frac{\bar{q}^r}{\bar{q}^R} \Sigma\right)}{(1 + \Sigma) Q^T - \kappa_1 \Sigma - q_1 \left(1 + \Sigma\right)}\right). (B158)
\]

**Case 9:** $\bar{q}^r > \bar{q}^R$, $q_1 < \bar{q}^R < \bar{q}^r \leq q_2$.

In this case, we have $R_1, A_2 < 0$, and $R_2, A_2 \geq 0$. Then,
\[
\frac{\bar{R} + R_1}{1 + A_1} = \frac{\bar{R} - |R_1|}{1 - |A_1|}. (B159)
\]
Calculating so that the optimal combination of $Y_1$ and $Y_2$ is $(1,0)$, corresponding to the optimal RVI values $\hat{r}_1 = T_h$, $\hat{r}_2 = T_l$.

Suppose now that $\bar{R} < \frac{|R_2|}{|A_2|}$. Then,

$$\bar{R} - \frac{\bar{R} - |R_1|}{1 - |A_1|} \leq 0,$$

$$\bar{R} - \frac{\bar{R} + |R_2|}{1 + |A_2|} \leq 0,$$

$$\frac{\bar{R} - |R_1|}{1 - |A_1|} \frac{\bar{R} - |R_1| + |R_2|}{1 - |A_1| + |A_2|} = \frac{|A_1| \left( \frac{|R_1|}{|A_1|} - \frac{\bar{R} + |R_2|}{1 + |A_2|} \right)}{1 - |A_1| + |A_2|},$$

$$= \frac{|A_1| \left( \frac{|R_1|}{|A_1|} - \bar{R} + \bar{R} - \frac{\bar{R} + |R_2|}{1 + |A_2|} \right)}{1 - |A_1| + |A_2|} \leq 0.$$  

Calculating

$$\frac{\bar{R} - |R_1|}{1 - |A_1|} \frac{\bar{R} - |R_1| + |R_2|}{1 - |A_1| + |A_2|} = \frac{|A_2| \left( \frac{\bar{R} - |R_1|}{1 - |A_1|} - \frac{\bar{R} + |R_2|}{1 + |A_2|} \right)}{1 - |A_1| + |A_2|}.$$
and using (B78), we observe that the optimal combination of $Y_1$ and $Y_2$ is $(1, 0)$ for $\tilde{R} < \frac{R_2}{|A_2|} \leq \tilde{R}^* = \frac{R - |R_1|}{1 - |A_1|}$ and $(1, 1)$ for $1 \leq \tilde{R} \leq \tilde{R}^* < \frac{R_2}{|A_2|}$. Thus, the optimal RVI values are $\hat{r}_1 = T_k$, $\hat{r}_2 = T_i$ for $\tilde{R} < \frac{R_2}{|A_2|} \leq \tilde{R}^*$ and $\hat{r}_1 = \hat{r}_2 = T_k$ for $1 \leq \tilde{R} \leq \tilde{R}^* < \frac{R_2}{|A_2|}$.

Note that $\tilde{R} < \frac{R_2}{|A_2|}$ is equivalent to

$$\frac{q_1}{q_2} - 1 > \tilde{R},$$

(B173)

or

$$\tilde{R} - 1 > \frac{q_2}{q_1} \left( \tilde{R} - \frac{q_3}{q_2} \right),$$

(B174)

and $\tilde{R}^* > \frac{R_2}{|A_2|}$ is equivalent to

$$\tilde{R}^* - 1 > \frac{q_2}{q_1} \left( \tilde{R}^* - \frac{q_3}{q_2} \right).$$

(B175)

In a similar manner, $\tilde{R} > \frac{R_1}{|A_1|}$ for $q_1 < \tilde{q}^R < \tilde{q}^R$ is equivalent to

$$\tilde{R} - 1 > \frac{q_1}{\tilde{q}^R} \left( \tilde{R} - \frac{\tilde{q}^R}{q_1} \right).$$

(B176)

Given that $\tilde{R} \leq \tilde{R}^*$, we have to consider three separate scenarios.

Under the first scenario, we have $1 < \frac{\tilde{q}^R}{q_1} < \tilde{R} \leq \tilde{R}^*$, and

$$\tilde{R}^* - 1 = \frac{\tilde{R} - |R_1|}{1 - |A_1|} - 1 = \frac{\tilde{R} - 1 - |R_1| + |A_1|}{1 - |A_1|},$$

(B177)

and

$$\tilde{R}^* - \frac{q_3}{q_2} = \frac{\tilde{R} - |R_1|}{1 - |A_1|} - \frac{q_3}{q_2} = \frac{\tilde{R} - \frac{q_3}{q_2} - |R_1| + \frac{q_3}{q_2} |A_1|}{1 - |A_1|},$$

(B178)

so that

$$\frac{\tilde{R}^* - 1}{\frac{q_3}{q_2}} = \frac{\tilde{R} - 1 - |R_1| + |A_1|}{\tilde{R} - \frac{q_3}{q_2} - |R_1| + \frac{q_3}{q_2} |A_1|} = \frac{Q^T - q_1 (1 + \frac{q_3}{q_2} \Sigma)}{Q^T - q_1 (1 + \Sigma) Q^T - \kappa_1 \Sigma - q_1 (1 + \Sigma)},$$

(B179)

with

$$\Sigma = \left( 1 - \frac{\tilde{q}^R}{q_1} \right) \frac{\delta R^R}{(1 - \delta) R^R T_i} < 0,$$

(B180)

for $\tilde{q}^R > q^R$. Thus, under this scenario ($\tilde{R} > \frac{\tilde{q}^R}{q_1}$), we have

$$q_2 < \tilde{q}^R \left( \frac{Q^T - q_1 (1 + \frac{q_3}{q_2} \Sigma)}{(1 + \Sigma) Q^T - \kappa_1 \Sigma - q_1 (1 + \Sigma)} \right),$$

(B181)
for (B175), and
\[
q_1 < \bar{q}^* \frac{\bar{R} - 1}{\bar{R} - \frac{q_1}{\bar{q}}} = \bar{q}^*.
\]  
(B182)

for (B174), with \(-1 < \Sigma < 0\). (B182) is satisfied for any \(q_1 < \bar{q}^R < \bar{q}^*\). Note that the right-hand side of (B181) is a decreasing function of \(q_1\), and the minimum of that expression on \(q_1 \in [0, \bar{q}^R]\) is reached at \(q_1 = \bar{q}^R\). This minimum is equal to
\[
\frac{Q^T - \bar{q}^R \left( 1 + \frac{\kappa_1}{\bar{q}^R} \right)}{(1 + \Sigma) Q^T - \Sigma - \bar{q}^R (1 + \Sigma)}.
\]  
(B183)

Then, since for \(\Sigma < 0\), (B183) is a decreasing function of \(\kappa_1\), we have its smallest value for \(\kappa_1 = 1\):
\[
\frac{Q^T - \bar{q}^R \left( 1 + \frac{1}{\bar{q}^R} \right)}{(1 + \Sigma) Q^T - \Sigma - \bar{q}^R (1 + \Sigma)}.
\]  
(B184)

Given that \(1 + \Sigma > 0\), and \(Q^T > 1\), we have \((1 + \Sigma) Q^T - \Sigma > 1 > \bar{q}^*\), so that (B184) is an increasing function of \(\bar{q}^R\). Taking the smallest possible value of \(\bar{q}^R = 0\), we get a lower bound on (B184):
\[
\frac{Q^T}{(1 + \Sigma) Q^T - \bar{q}^*}.
\]  
(B185)

Since for \(\Sigma < 0\) (B185) is an increasing function of \(Q^T\), the smallest possible value of (B185) is obtained for \(Q^T = 1\) and is equal to 1. Thus, the right-hand side of (B181) is greater than or equal to \(\bar{q}^*\).

Under the second scenario, we have \(1 < \bar{R} \leq \frac{\bar{u}}{\bar{q}} \leq \bar{R}^*\), and (B174) and (B176) are satisfied for any \(q_1\) and \(q_2\). If \(\frac{\bar{u}}{\bar{q}} = \bar{R}^*\), then (B175) is also satisfied for any \(q_2\). If \(\frac{\bar{u}}{\bar{q}} < \bar{R}^*\), then (B175) becomes
\[
q_2 < \bar{q}^* \left( \frac{\bar{R}^* - 1}{\bar{R}^* - \frac{\bar{u}}{\bar{q}}} \right).
\]  
(B186)

Using (B179), we have
\[
\frac{\bar{R}^* - 1}{\bar{R}^* - \frac{\bar{u}}{\bar{q}}} = \frac{\bar{R} - 1 - \frac{q_1}{\bar{q}}}{\bar{R} - \frac{q_1}{\bar{q}}} \left( \frac{\kappa_1}{Q^T - q_1} \right) \left( 1 - \frac{\bar{q}^*}{\bar{q}} \right).
\]  
(B187)

If \(\frac{\bar{u}}{\bar{q}} = \bar{R}\), we get
\[
\frac{\bar{R}^* - 1}{\bar{R}^* - \frac{\bar{u}}{\bar{q}}} = \frac{\bar{u} - \frac{q_1}{\bar{q}}}{\bar{R} - \frac{q_1}{\bar{q}}} \left( \frac{\kappa_1}{Q^T - q_1} \right) \left( 1 - \frac{\bar{q}^*}{\bar{q}} \right) = \frac{1 + \frac{\bar{u}}{\bar{q}} \left( \frac{\kappa_1}{Q^T - q_1} \right)}{\left( \frac{\kappa_1}{Q^T - q_1} \right) \left( 1 - \frac{\bar{q}^*}{\bar{q}} \right)}.
\]  
(B188)

Note that in this case, formally, \(\Sigma = -1\). For \(\frac{\bar{u}}{\bar{q}} > \bar{R}\), we have \(\Sigma < -1\), but we can still use (B181):
\[
q_2 < \bar{q}^* \left( \frac{Q^T - q_1 \left( 1 + \frac{\kappa_1}{\bar{q}} \right)}{(1 + \Sigma) Q^T - \kappa_1 \Sigma - q_1 (1 + \Sigma)} \right),
\]  
(B189)

which we can re-express as
\[
q_2 < \bar{q}^* \left( \frac{Q^T - q_1 \left( 1 - \frac{\kappa_1}{\bar{q}} \right) \left| \Sigma \right|}{(1 - \left| \Sigma \right|) Q^T + \kappa_1 \left| \Sigma \right| - q_1 (1 - \left| \Sigma \right|)} \right).
\]  
(B190)
As the right-hand side of (B190) is an increasing function of $Q^T$, its lowest value is achieved for $Q^T = 1$, and is given by
\[
1 - q_1 \left( 1 - \frac{q}{\bar{q}} |\Sigma| \right) \frac{(1 - |\Sigma|) + \kappa_1 |\Sigma| - q_1 (1 - |\Sigma|)}{1 - q_1 (1 - |\Sigma|)}.
\]
(B191)

The right-hand side of (B191) is a decreasing function of $\kappa_1$, and by setting $\kappa_1 = 1$, we get the lowest possible value
\[
1 - q_1 \left( 1 - \frac{q}{\bar{q}} |\Sigma| \right),
\]
(B192)

which is an increasing function of $q_1$, reaching its minimum, $\bar{q}^*$ at $q_1 = 0$. Thus, the right-hand side of (B189) is greater than or equal to $\bar{q}^*$.

Finally, under the third scenario, $\bar{R} \leq \bar{R}^* < \frac{q}{\bar{q}}$, and (B174)-(B176) are satisfied for any $q_1$ and $q_2$. This is expressed using the $\max(\cdot, 0) = (\cdot)^+$ operator in (B11).

Case 10: $\bar{q}^* > q^R$, $q^R < q_1 < q_2 < \bar{q}^*$.

In this case, we have $R_1, R_2 > 0$, and $A_1, A_2 < 0$. Then,
\[
\frac{\bar{R} + R_1}{1 + A_1} = \frac{\bar{R} + |R_1|}{1 - |A_1|},
\]
(B193)

\[
\frac{\bar{R} + R_2}{1 + A_2} = \frac{\bar{R} + |R_2|}{1 - |A_2|},
\]
(B194)

\[
\frac{\bar{R} + R_1 + R_2}{1 + A_1 + A_2} = \frac{\bar{R} + |R_1| + |R_2|}{1 - |A_1| - |A_2|},
\]
(B195)

and the optimal combination of $Y_1$ and $Y_2$ is $(1, 1)$, corresponding to the optimal RVI values $\hat{r}_1 = T_h, \hat{r}_2 = T_h$.

Case 11: $\bar{q}^* > q^R$, $q^R < q_1 < \bar{q}^* \leq q_2$.

In this case, we have $R_1, R_2, A_2 > 0$, $A_1 < 0$, and $|r_2| \geq 1$. Then,
\[
\frac{\bar{R} + R_1}{1 + A_1} = \frac{\bar{R} + |R_1|}{1 - |A_1|},
\]
(B196)

\[
\frac{\bar{R} + R_2}{1 + A_2} = \frac{\bar{R} + |R_2|}{1 + |A_2|},
\]
(B197)

\[
\frac{\bar{R} + R_1 + R_2}{1 + A_1 + A_2} = \frac{\bar{R} + |R_1| + |R_2|}{1 - |A_1| + |A_2|},
\]
(B198)

so that the optimal value for $Y_1$ is 1. The optimal value of $Y_2$ is determined by the relative values of $\frac{R_2}{R_1} |A_1|^{-1}$ and $\frac{R_2}{R_1} |A_1|^{-1}$. Then, we have
\[
\frac{\bar{R} + |R_1|}{1 - |A_1|} - \frac{\bar{R} + |R_1| + |R_2|}{1 - |A_1| + |A_2|} = \frac{|A_2| (R_2 + |R_1|) - |r_2|}{1 - |A_1| + |A_2|} \geq 0
\]
(B199)

if and only if
\[
\frac{|r_2|}{|A_2|} \leq \frac{\bar{R} + |R_1|}{1 - |A_1|}.
\]
(B200)
Note that, for $\bar{q}^R < q_1 < \bar{q}^\tau \leq q_2$, we have $\bar{R}^*$, defined in (B78), equal to
\[ \bar{R}^* = \frac{\bar{R} + \mathcal{R}_1}{1 + \mathcal{A}_1} = \frac{\bar{R} + |\mathcal{R}_1|}{1 - |\mathcal{A}_1|}. \] (B201)

Thus we have the optimal combination of $Y_1$ and $Y_2$ to be $(1, 0)$ (corresponding to the optimal RVI values $\hat{r}_1 = T_h$, $\hat{r}_2 = T_l$) for $\frac{|\mathcal{R}_2|}{|\mathcal{A}_2|} \leq \bar{R}^*$, and $(1, 1)$ (corresponding to the optimal RVI values $\hat{r}_1 = \hat{r}_2 = T_h$) for $|\mathcal{R}_2| > \bar{R}^*$. Note that in this case, $|\mathcal{R}_2| > \bar{R}^*$ is equivalent to
\[ \bar{R} - \frac{q_2}{\bar{q}^\tau} \left( \frac{\bar{R} - 1}{\bar{R} - \frac{q^\tau}{q^R}} \right), \] (B202)
which is the same as (B11).

Case 12: $\bar{q}^\tau > q^R$, $\bar{q}^R < \bar{q}^\tau \leq q_1 < q_2$.

In this case, we have $\mathcal{R}_1, \mathcal{R}_2, \mathcal{A}_1, \mathcal{A}_2 > 0$. Then,
\[ \frac{\bar{R} + \mathcal{R}_1}{1 + \mathcal{A}_1} = \frac{\bar{R} + |\mathcal{R}_1|}{1 + |\mathcal{A}_1|}, \] (B204)
\[ \frac{\bar{R} + \mathcal{R}_2}{1 + \mathcal{A}_2} = \frac{\bar{R} + |\mathcal{R}_2|}{1 + |\mathcal{A}_2|}, \] (B205)
\[ \frac{\bar{R} + \mathcal{R}_1 + \mathcal{R}_2}{1 + \mathcal{A}_1 + \mathcal{A}_2} = \frac{\bar{R} + |\mathcal{R}_1| + |\mathcal{R}_2|}{1 + |\mathcal{A}_1| + |\mathcal{A}_2|}, \] (B206)

Note that for $\bar{q}^R < \bar{q}^\tau \leq q_1 < q_2$ we have $\frac{|\mathcal{R}_1|}{|\mathcal{A}_1|} > \frac{|\mathcal{R}_2|}{|\mathcal{A}_2|} \geq 1$.

Suppose that $\bar{R} \geq \frac{|\mathcal{R}_1|}{|\mathcal{A}_1|}$, which is equivalent to
\[ \frac{\bar{q}^R - 1}{\bar{q}^\tau - 1} \leq \bar{R}, \] (B207)
or
\[ \frac{q_1}{\bar{q}^\tau} \left( \bar{R} - \frac{\bar{q}^\tau}{\bar{q}^R} \right) > \bar{R} - 1, \] (B208)
which, in turn, implies $\bar{R} - \frac{\bar{q}^\tau}{\bar{q}^R} > 0$ and
\[ q_1 > \bar{q}^\tau \left( \frac{\bar{R} - 1}{\bar{R} - \frac{q^\tau}{q^R}} \right) = \frac{\bar{q}^\tau}{1 + \Sigma}, \] (B209)
with $1 + \Sigma > 0$.

Then,
\[ \bar{R} \geq \frac{\bar{R} + |\mathcal{R}_1|}{1 + |\mathcal{A}_1|}, \] (B210)
\[ \bar{R} > \frac{\bar{R} + |\mathcal{R}_2|}{1 + |\mathcal{A}_2|}, \] (B211)
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\[
\hat{R} > \frac{\hat{R} + |R_1| + |R_2|}{1 + |A_1| + |A_2|},
\]

so that the optimal combination of \(Y_1\) and \(Y_2\) is \((0, 0)\), corresponding to the optimal RVI values \(\hat{r}_1 = \hat{r}_2 = T_i\).

Now, suppose that \(\frac{|R_2|}{|A_2|} < \hat{R} < \frac{|R_1|}{|A_1|}\). Similar to (B208), this implies

\[
q_2 > \frac{\hat{q}'}{1 + \Sigma},
\]

with \(1 + \Sigma > 0\), and

\[
q_1 \leq \frac{\hat{q}'}{1 + \Sigma},
\]

Then,

\[
\hat{R} < \frac{\hat{R} + |R_1|}{1 + |A_1|},
\]

\[
\hat{R} < \frac{\hat{R} + |R_2|}{1 + |A_2|},
\]

and

\[
\frac{\hat{R} + |R_1| + |R_2|}{1 + |A_1| + |A_2|} - \frac{\hat{R} + |R_1|}{1 + |A_1|} = \frac{|A_2| \left( \left| \frac{|R_2|}{|A_2|} - \frac{\hat{R} + |R_2|}{1 + |A_2|} \right) \right)}{1 + |A_1| + |A_2|}
\]

\[
\leq 0,
\]

so that the optimal combination of \(Y_1\) and \(Y_2\) is \((1, 0)\), corresponding to the optimal RVI values \(\hat{r}_1 = T_h\), \(\hat{r}_2 = T_l\).

Finally, suppose that \(1 \leq \hat{R} < \frac{|R_2|}{|A_2|}\), which implies

\[
q_1 \leq \frac{\hat{q}'}{(1 + \Sigma)^+},
\]

\[
q_2 \leq \frac{\hat{q}'}{(1 + \Sigma)^+}.
\]

Then,

\[
\hat{R} < \frac{\hat{R} + |R_1|}{1 + |A_1|},
\]

\[
\hat{R} < \frac{\hat{R} + |R_2|}{1 + |A_2|},
\]

Further,

\[
\frac{\hat{R} + |R_1| + |R_2|}{1 + |A_1| + |A_2|} - \frac{\hat{R} + |R_2|}{1 + |A_2|} = \frac{|A_1| \left( \left| \frac{|R_1|}{|A_1|} - \frac{\hat{R} + |R_2|}{1 + |A_2|} \right) \right)}{1 + |A_1| + |A_2|}
\]

\[
= \frac{|A_1| \left( \left| \frac{|R_1|}{|A_1|} - \frac{\hat{R} + |R_2|}{1 + |A_2|} \right) \right)}{1 + |A_1| + |A_2|} \geq \frac{|A_1| \left( \left| \frac{|R_2|}{|A_2|} - \frac{\hat{R} + |R_2|}{1 + |A_2|} \right) \right)}{1 + |A_1| + |A_2|}.
\]
\[ \frac{|A_1| \left( \frac{|R_2|}{|A_2|} - \bar{R} \right)}{(1 + |A_2|) (1 + |A_1| + |A_2|)} > 0, \quad (B223) \]

and

\[ \frac{\bar{R} + |R_1| + |R_2|}{1 + |A_1| + |A_2|} - \frac{\bar{R} + |R_1|}{1 + |A_1|} = \frac{|A_2| \left( \frac{|R_2|}{|A_2|} - \frac{R + |R_1|}{1 + |A_1|} \right)}{1 + |A_1| + |A_2|}. \quad (B224) \]

This last expression is non-negative if and only if \( \frac{|R_2|}{|A_2|} \geq \bar{R}^* \). Thus, the optimal combination of \( Y_1 \) and \( Y_2 \) is (1, 1), and the optimal RVI values are \( \hat{r}_1 = \hat{r}_2 = T_h \) for \( \frac{|R_2|}{|A_2|} > \bar{R}^* \), and (1, 0), and the optimal RVI values are \( \hat{r}_1 = T_h, \hat{r}_2 = T_l \) for \( \frac{|R_2|}{|A_2|} \leq \bar{R}^* \). Note that in this case, \( \frac{|R_2|}{|A_2|} > \bar{R}^* \) is equivalent to

\[ \bar{R}^* - 1 > \frac{q_2}{q^*} \left( \frac{\bar{R}^* - \bar{q}^R}{\bar{q}^R} \right), \quad (B225) \]

similar to (B202). (B225) is, in turn, equivalent to

\[ q_2 < \frac{1}{\left( \frac{R^* - R}{R - \bar{q}^R} \right)^\gamma}, \quad (B226) \]

which is the same as (B11).

**Proof of Corollary B1**

In the setting with \( \bar{q}^R = \bar{q}^* \), we have

\[ \frac{R_i}{A_i} = 1, i = 1, 2. \quad (B227) \]

Note that \( \bar{R} = 1 + \frac{(1 - \delta) K R_1}{\delta R^*} > 1 = \frac{R_i}{A_i}, i = 1, 2. \) Then, as it follows from (B10) and (B11), \( \hat{r}_1 = T_h \) whenever \( q_1 \leq \bar{q}^* \), and \( \hat{r}_1 = T_l \) otherwise, and \( \hat{r}_2 = T_h \) whenever \( q_2 \leq \bar{q}^* \), and \( \hat{r}_2 = T_l \) otherwise.

**Proof of Proposition B3**

Under the proportional compensation, \( \Sigma = 0 \), and, as shown in Proposition B1, the setting where different RVIs can be applied to different patient groups and the setting where the same RVI is used result in different optimal choices, if

\[ q_1 \leq \bar{q}^*, \quad (B228) \]

and

\[ q_2 > \bar{q}^*. \quad (B229) \]

In particular, under (B228) and (B229), if the RVI value can be set individually for each patient groups, the optimal RVI for group 1 is \( \hat{r}_1 = T_h \), while the optimal value RVI for group 2 is \( \hat{r}_2 = T_l \). On the other
hand, the optimal RVI value for the setting where a single RVI is applied to both groups depends on the value of \( \kappa_1 \), resulting in two cases.

Case 1: \( \kappa_1 > \hat{\kappa}_1 \).

In the "uniform" RVI case, the optimal RVI value to apply to both patient groups is \( \hat{r} = T_h \), so that the physician's revenue is given by

\[
R_s = \frac{(1 - \delta) R^d + \delta \left( \kappa_1 \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} + \kappa_2 \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)}{\kappa_1 \left( \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} \right) + \kappa_2 \left( \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)}
\]

\[
= (1 - \delta) R^d + \delta R^c \left( \kappa_1 \left( 1 + \frac{q_1 (\frac{u}{r} - 1)}{1 + q_1 (\frac{u}{r} - 1)} \right) + \kappa_2 \left( \frac{1 + q_2 (\frac{u}{r} - 1)}{1 + q_2 (\frac{u}{r} - 1)} \right) \right)
\]

\[
= \frac{\delta R^c}{\tau^r} \left( 1 + \frac{(1 - \delta) R^d T_l}{\delta R^c} \right) \left( 1 + \frac{1}{\kappa_1 \left( \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} \right) + \kappa_2 \left( \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)} \right)
\]

\[
= \frac{\delta R^c}{\tau^r} \left( 1 + \frac{(1 - \delta) R^d T_l}{\delta R^c} \right) \left( 1 + \frac{1}{\kappa_1 \left( \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} \right) + \kappa_2 \left( \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)} \right)
\]

\[
= \frac{\delta R^c}{\tau^r} \left( 1 + \frac{(1 - \delta) R^d T_l}{\delta R^c} \right) \left( 1 + \frac{1}{\kappa_1 \left( \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} \right) + \kappa_2 \left( \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)} \right)
\]

\[
\text{where we have used } \frac{u}{r} = \frac{R^c}{R^d} \text{ and } \frac{u}{r} - 1 = \left( \frac{T_h}{T_l} - 1 \right) \left( \frac{1}{q_r} - 1 \right).
\]

Note that for \( q_1 \leq q_r < q_2 \) we have

\[
1 + q_1 \left( \frac{1}{q_r} - 1 \right) \left( \frac{T_h}{T_l} - 1 \right) \leq 1 + q_2 \left( \frac{1}{q_r} - 1 \right) \left( \frac{T_h}{T_l} - 1 \right) \leq 1 + q_2 \left( \frac{1}{q_r} - 1 \right) \left( \frac{T_h}{T_l} - 1 \right).
\]

In the setting where \( \hat{r}_1 = T_h \) and \( \hat{r}_2 = T_l \), the physician's revenue is

\[
R_d = \frac{\delta R^c}{\tau^r} \left( 1 + \frac{(1 - \delta) R^d T_l}{\delta R^c} \right) \left( 1 + \frac{1}{\kappa_1 \left( \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} \right) + \kappa_2 \left( \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)} \right)
\]

Therefore, the ratio of the two revenue values is

\[
\frac{R_s}{R_d} = \frac{1 + \left( \frac{(1-\delta) R^d T_l}{\delta R^c} \right) \left( \frac{1}{\kappa_1 \left( \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} \right) + \kappa_2 \left( \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)} \right)}{1 + \left( \frac{(1-\delta) R^d T_l}{\delta R^c} \right) \left( \frac{1}{\kappa_1 \left( \frac{(1-q_1) R^c + q_1 R_n}{(1-q_1) T_h + q_1 T_l} \right) + \kappa_2 \left( \frac{(1-q_2) R^c + q_2 R_n}{(1-q_2) T_h + q_2 T_l} \right)} \right)}
\]

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where $1 - \frac{R_s}{R_d}$ provides the expression in (B17).

We would like to show that this ratio is an increasing function of $\kappa_1$ for $\kappa_1 \geq \bar{\kappa}_1$. Note that $R_s \leq R_d$, and

$$\frac{\partial R_s}{\partial \kappa_1} = \frac{\partial R_d}{\partial \kappa_1} = \frac{\partial R_s}{\partial \kappa_1} R_d - \frac{\partial R_d}{\partial \kappa_1} R_s$$

will be positive as long as

$$\frac{\partial R_s}{\partial \kappa_1} > \frac{\partial R_d}{\partial \kappa_1}.$$  \hspace{1cm} (B236)

Denoting

$$B(q) = \frac{1 + q \left( \frac{1}{q} - 1 \right) \left( \frac{T_2}{T_1} - 1 \right)}{1 + q \left( \frac{1}{q} - 1 \right) \left( \frac{T_2}{T_1} - 1 \right)},$$

we get

$$\frac{\partial R_s}{\partial \kappa_1} = \left( 1 - \delta \right) R^d T_1 \left( \frac{B(q_2) - B(q_1)}{(B(q_2) + \kappa_1 (B(q_1) - B(q_2)))^2} \right),$$ \hspace{1cm} (B238)

$$\frac{\partial R_d}{\partial \kappa_1} = \left( 1 - \delta \right) R^d T_1 \left( \frac{1 - B(q_1)}{(1 + \kappa_1 (B(q_1) - 1))^2} \right),$$ \hspace{1cm} (B239)

where $B(q_2) > 1 \geq B(q_1)$. We observe that (B236) holds if and only if

$$\left( \frac{B(q_2) - B(q_1)}{(B(q_2) + \kappa_1 (B(q_1) - B(q_2)))^2} \right) > \left( \frac{1 - B(q_1)}{(1 + \kappa_1 (B(q_1) - 1))^2} \right),$$ \hspace{1cm} (B240)

or

$$\frac{B(q_2) - \kappa_1 (B(q_2) - B(q_1))}{\sqrt{B(q_2) - B(q_1)}} < \frac{1 - \kappa_1 (1 - B(q_1))}{\sqrt{1 - B(q_1)}}.$$ \hspace{1cm} (B241)

Note that (B241) can be expressed as

$$\kappa_1 > \frac{B(q_2) - B(q_1)}{\sqrt{B(q_2) - B(q_1)}} - \frac{1}{\sqrt{1 - B(q_1)}}.$$ \hspace{1cm} (B242)

Note that $\bar{\kappa}_1$ satisfies

$$B(q_2) + \bar{\kappa}_1 (B(q_1) - B(q_2)) = 1,$$ \hspace{1cm} (B243)

so that for any $\kappa_1 > \bar{\kappa}_1$, we have

$$B(q_2) + \kappa_1 (B(q_1) - B(q_2)) < 1,$$ \hspace{1cm} (B244)

or

$$\kappa_1 > \frac{B(q_2) - 1}{B(q_2) - B(q_1)}.$$ \hspace{1cm} (B245)

Thus, for

$$\frac{B(q_2) - 1}{B(q_2) - B(q_1)} > \frac{B(q_2) - B(q_1)}{\sqrt{B(q_2) - B(q_1)} - \sqrt{1 - B(q_1)}} - \frac{1}{\sqrt{1 - B(q_1)}},$$

$$= \frac{B(q_2) - 1 + 1 - \sqrt{B(q_2) - B(q_1)}}{B(q_2) - B(q_1) - \sqrt{B(q_2) - B(q_1)} \sqrt{1 - B(q_1)}}.$$ \hspace{1cm} (B246)
the ratio of the two revenue values is an increasing function of $\kappa_1$. Defining

$$A = B(q_2) - 1,$$

$$C = B(q_2) - B(q_1),$$

$$X = \sqrt{\frac{B(q_2) - B(q_1)}{1 - B(q_1)}} - 1,$$

$$Y = \sqrt{(B(q_2) - B(q_1))(1 - B(q_1))},$$

we can re-express (B246) as

$$\frac{A}{C} = \frac{A - X}{C - Y},$$

that holds if and only if

$$\frac{X}{Y} \geq \frac{A}{C},$$

or

$$\frac{B(q_2) - 1}{B(q_2) - B(q_1)} \leq \frac{\sqrt{\frac{B(q_2) - B(q_1)}{1 - B(q_1)}} - 1}{\sqrt{(B(q_2) - B(q_1))(1 - B(q_1))}}.$$  \hspace{1cm} (B253)

Using

$$D = 1 - B(q_1),$$

we have for (B253)

$$\frac{A}{A + D} \leq \frac{\sqrt{A + D} - 1}{\sqrt{(A + D)(D)}},$$

or

$$\frac{AD}{A + D} \leq 1 - \sqrt{\frac{D}{A + D}}.$$  \hspace{1cm} (B256)

Further, (B256) is equivalent to

$$\frac{AD}{A + D} + \sqrt{\frac{D}{A + D}} \leq 1.$$  \hspace{1cm} (B257)

The left-hand side of (B257) is an increasing function of $D$. Given that $D = 1 - B(q_1)$, the maximum value of $D$ is realized for the minimum value of $B(q_1)$. Note that

$$B(q_1) = \frac{1 + q_1 \left( \frac{T}{T_f} - 1 \right) \left( \frac{T}{T_f} - 1 \right)}{1 + (1 - q_1) \left( \frac{T}{T_f} - 1 \right)},$$

is an increasing function of $q_1$, reaching minimum at $q_1 = 0$. The value of this minimum is $\frac{T}{T_f}$, resulting in the maximum value of $D$ to be $D_m = 1 - \frac{T}{T_f}$. Then, if (B257) were to hold for any $D \in \left[ 0, 1 - \frac{T}{T_f} \right]$, we will have

$$\frac{AD_m}{A + D_m} + \sqrt{\frac{D_m}{A + D_m}} = D_m - D_m \left( \sqrt{\frac{D_m}{A + D_m}} \right)^2 + \sqrt{\frac{D_m}{A + D_m}} \leq 1.$$  \hspace{1cm} (B259)
Given that, for \( q_2 \in [\bar{q}, 1] \), the value of \( A = B(q_2) - 1 \) is between 0 and

\[
A_m = \left( \frac{1}{\bar{q}} - 1 \right) \left( \frac{T_h}{T_l} - 1 \right),
\]

the maximum of

\[
D_m - D_m t^2 + t
\]

with

\[
t = \sqrt{\frac{D_m}{A + D_m}} \leq 1,
\]

is reached either at \( t = 1 \) or at \( t = \frac{1}{2D_m} \), whichever is smaller. Note that the value of (B261) for \( t = 1 \) is 1, so, in order for \( D_m - D_m t^2 + t \leq 1 \) to hold we must require that

\[
\frac{1}{2D_m} \geq 1,
\]

which is the same as \( T_l \geq \frac{1}{2} T_h \).

Case 2: \( \kappa_1 \leq \hat{\kappa}_1 \).

In the “uniform” RVI case, \( \hat{r} = T_h \), and the physician’s revenue is given by

\[
R_s = \left( 1 - \delta \right) R_d + \delta \frac{R^c}{T_l} = \delta \frac{R^c}{r^c} \left( 1 + \left( \frac{1 - \delta}{\delta R^c} T_h \right) \right).
\]

In the heterogeneous RVI case, \( \hat{r}_1 = T_h \), \( \hat{r}_2 = T_l \), and the physician’s revenue is given by (B233). The ratio of these two revenue values of revenue is

\[
\frac{R_s}{R_d} = 1 + \frac{(1 - \delta) R_d T_l}{\delta R^c} \cdot \left( 1 + \frac{1 + q_1 (\frac{1}{\hat{r}_1} - 1)}{1 + q_1 (\frac{1}{\hat{r}_2} - 1)} \right)^{-1},
\]

which, for \( q_1 < \bar{q} \) is a decreasing function of \( \kappa_1 \). Also, \( 1 - \frac{R_s}{R_d} \) provides the expression in (B16).

As the results of both Case 1 and Case 2 imply, the ratio of the two revenue values is bound from below by the value achieved at \( \kappa_1 = \hat{\kappa}_1 \). Then the relative performance gap of the “uniform” RVI approach when applied to the two-patient-group setting cannot exceed

\[
e^S = 1 - \frac{1 + \frac{(1 - \delta) R_d T_l}{\delta R^c}}{1 + \frac{(1 - \delta) R_d T_l}{\delta R^c} \cdot \left( 1 + \frac{1 + q_1 (\frac{1}{\hat{r}_1} - 1)}{1 + q_1 (\frac{1}{\hat{r}_2} - 1)} \right)^{-1}} = \hat{\kappa}_1 \left( 1 + \frac{(1 - \delta) R_d T_l}{\delta R^c} \cdot \left( 1 - \frac{1 + q_1 (\frac{1}{\hat{r}_1} - 1)}{1 + q_1 (\frac{1}{\hat{r}_2} - 1)} \right) \right).
\]

\( \square \)
Proof of Proposition B4

Suppose that we are in a setting where patients choose to adopt e-visits.

a) The result follows from the proof of Proposition B2 with \( q^+ (c, \Delta), q^- (c, \Delta), q^R, R, \Sigma, i = 1, 2 \) replaced by \( q^+ (c, \Delta, R^e, \alpha^e, \beta^e), q^- (c, \Delta, R^e, \alpha^e, \beta^e), \tilde{q}^e, \tilde{q}^R, \tilde{R}^e, \Sigma_e, i = 1, 2 \), respectively.

b) Note that \( \tilde{\alpha}_e^e (q_2) \leq \alpha^e_e \) implies, according to the result of part b) of Proposition 3, that the patient group 2 is inflexible. In particular, \( q_2 > q^e_e (c, \Delta, R^e_e, \alpha^e_e, \beta^e_e) \) and \( \tilde{r}_2 = T_i \). On the other hand, \( \alpha^e_e \leq \tilde{\alpha}_e^e (q_1) \), the patient group 1 remains flexible, since \( q^-_e (c, \Delta, R^e_e, \alpha^e_e, \beta^e_e) < q_1 < q^e_e (c, \Delta, R^e_e, \alpha^e_e, \beta^e_e) \). The choice of the RVI value for the patient group 1 is dictated by the following version of (38)-(40):

\[
\begin{align*}
\max_{N, \tau_1} & \left( N \left( (1-\delta) \tilde{R}_e^e + \delta \kappa_1 \left( \frac{\rho^e_1 (r_1) \tilde{R}_e^e + (1-\rho^e_1 (r_1)) R^u}{T_1 (r_1)} \right) \right) + \delta \kappa_2 \left( \frac{\tilde{R}_e^e}{T_1} \right) \right) \tag{B267} \\
\text{s.t.} & \ N \left( \kappa_1 \left( \frac{\rho^e_1 (r_1) \tilde{e}^e + (1-\rho^e_1 (r_1)) \tau^u}{T_1 (r_1)} \right) + \kappa_2 \left( \frac{\tilde{e}^e}{T_1} \right) \right) \leq A, \tag{B268} \\
& \ r_1 \in \{ T_i, T_h \}. \tag{B269}
\end{align*}
\]

Then, introducing

\[
\begin{align*}
\mathcal{R}_e^e &= \kappa_e \left( \frac{\tilde{q}^e_e - \tilde{q}^R_e}{q^e_e (Q^e_1 - \tilde{q}^e_1)} \right), \quad i = 1, 2, \tag{B270} \\
\mathcal{A}_e^e &= \kappa_e \left( \frac{\tilde{q}^e_e - \tilde{\eta}^e_e}{q^e_e (Q^e_1 - \tilde{q}^e_1)} \right), \quad i = 1, 2, \tag{B271}
\end{align*}
\]

we can re-express (B267)-(B269) as

\[
\begin{align*}
\max_{Y_1} & \frac{\tilde{R}_e^e + \mathcal{R}_e^e Y_1}{1 + \mathcal{A}_e^e Y_1} \tag{B272} \\
\text{s.t.} & \ Y_1 \in \{0, 1\}, \tag{B273}
\end{align*}
\]

so that the best RVI value for the patient group 1 is determined by the relative values of \( \tilde{R}_e^e \) and \( \frac{\mathcal{R}_e^e + \mathcal{A}_e^e}{1 + \mathcal{A}_e^e} \). Since \( 1 + \mathcal{A}_e^e > 0 \),

\[
\tilde{R}_e^e \leq \frac{\tilde{R}_e^e + \mathcal{R}_e^e}{1 + \mathcal{A}_e^e} \Leftrightarrow \tilde{R}_e^e \mathcal{A}_e^e \leq \mathcal{R}_e^e \Leftrightarrow \tilde{R}_e^e \left( \frac{q_1}{\tilde{q}^e_e} - 1 \right) \leq \frac{q_1}{\tilde{q}^R_e} - 1. \tag{B274}
\]

Suppose that \( \tilde{q}^e_e \leq \tilde{q}^R_e \), so that \( \Sigma_e > 0 \), and consider three possible cases: \( q_1 \leq \tilde{q}^e_e \), \( \tilde{q}^e_e < q_1 \leq \tilde{q}^R_e \), and \( \tilde{q}^R_e \leq q_1 \).

In the first case, (B274) is equivalent to

\[
q_1 \leq \tilde{q}^e_e \left( \frac{1}{1 + \left( \frac{\tilde{q}^R_e}{\tilde{q}^e_e} \right)} \right) = \tilde{q}^e_e \left( \frac{1}{1 + \left( \frac{\delta \mathcal{R}_e^e (1-\delta) \tilde{R} e^e T_1}{\delta \mathcal{R}_e^e \tilde{R} e^e T_1} \right) \left( 1 - \frac{\tilde{q}^e_e}{\tilde{q}^R_e} \right) } \right) = \frac{\tilde{q}^e_e}{1 + \Sigma_e}. \tag{B275}
\]

Note that the expression on the right-hand side of (B275) is less or equal to \( \tilde{q}^e_e \). Thus, the optimal value of \( r_1 \) is equal to \( T_h \) if and only if (B275) holds.

Now, consider the case of \( \tilde{q}^e_e \leq q_1 \leq \tilde{q}^R_e \), and \( \tilde{q}^e_e < q_1 \leq \tilde{q}^R_e \). In this case, we have to compare \( \tilde{R} e^e \) and \( \frac{\tilde{R} e^e + \mathcal{R}_e^e}{1 + \mathcal{A}_e^e} \), and, given that \( \mathcal{R}_e^e < 0 \) and \( \mathcal{A}_e^e > 0 \), we have \( \tilde{R} e^e > \frac{\tilde{R} e^e + \mathcal{R}_e^e}{1 + \mathcal{A}_e^e} \), and the optimal value of \( r_1 \) is equal to \( T_i \).

Finally, for \( \tilde{q}^e_e \leq \tilde{q}^R_e \) and \( \tilde{q}^R_e < q_1 \), (B274) becomes

\[
\tilde{R} e^e \left( \frac{q_1}{\tilde{q}^e_e} - 1 \right) \leq \frac{q_1}{\tilde{q}^R_e} - 1 \Leftrightarrow \frac{q_1}{\tilde{q}^e_e} \leq \frac{1}{1 + \left( \frac{\delta \mathcal{R}_e^e (1-\delta) \tilde{R} e^e T_1}{\delta \mathcal{R}_e^e \tilde{R} e^e T_1} \right) \left( 1 - \frac{\tilde{q}^e_e}{\tilde{q}^R_e} \right) } = \frac{\tilde{q}^e_e}{1 + \Sigma_e}, \tag{B276}
\]
which does not hold for $\bar{q}_e^c > \bar{q}_e^R < q_1$. Thus, the optimal value of $r_1$ is equal to $T_h$.

Suppose now that $\bar{q}_e^c > \bar{q}_e^R$, so that $\Sigma_e < 0$, and consider three possible cases: $q_1 \leq \bar{q}_e^R$, $\bar{q}_e^R < q_1 \leq \bar{q}_e^c$, and $\bar{q}_e^c < q_1$.

In the first case, we have

$$\bar{R}^e \left( \frac{q_1}{q_e^c} - 1 \right) \leq \frac{q_1}{\bar{q}_e^R} - 1 \Leftrightarrow \bar{R}^e \left( 1 - \frac{q_1}{\bar{q}_e^R} \right) \geq 1 - \frac{q_1}{q_e^c},$$

which is the same as

$$\bar{R}^e - 1 \geq \frac{q_1}{\bar{q}_e^R} \left( \bar{R}^e - \frac{\bar{q}_e^c}{\bar{q}_e^R} \right).$$

Then, for $\bar{R}^e \leq \frac{\bar{q}_e^c}{\bar{q}_e^R}$, (B278) holds for any $q_1$. On the other hand, for $\bar{R}^e > \frac{\bar{q}_e^c}{\bar{q}_e^R}$, which is equivalent to

$$\left( \frac{\delta R_e^r}{(1 - \delta) R_e^r T_h} \right) \left( \frac{\bar{q}_e^c}{\bar{q}_e^R} - 1 \right) < 1,$$

(B279) holds if and only if

$$q_1 \leq \bar{q}_e^c \left( \frac{1}{1 - \left( \frac{\delta R_e^r}{(1 - \delta) R_e^r T_h} \right) \left( \frac{\bar{q}_e^c}{\bar{q}_e^R} - 1 \right)} \right).$$

Note, however, that

$$\bar{q}_e^c \left( \frac{1}{1 - \left( \frac{\delta R_e^r}{(1 - \delta) R_e^r T_h} \right) \left( \frac{\bar{q}_e^c}{\bar{q}_e^R} - 1 \right)} \right) - \bar{q}_e^R$$

$$= \bar{q}_e^c \left( \frac{\bar{q}_e^c}{\bar{q}_e^R} - 1 \right) \left( \frac{\delta R_e^r}{(1 - \delta) R_e^r T_h} \right) + 1$$

$$= \bar{q}_e^c \left( \frac{\bar{q}_e^c}{\bar{q}_e^R} - 1 \right) \left( \frac{\delta R_e^r}{(1 - \delta) R_e^r T_h} \right) + 1 > 0,$$

so that (B280) holds for any $q_1 < \bar{q}_e^R$. Thus, (B278) holds for any $q_1 < \bar{q}_e^R$ and the optimal value of $r_1$ is equal to $T_h$.

Next, consider $\bar{q}_e^c > \bar{q}_e^R$ and $\bar{q}_e^R < q_1 \leq \bar{q}_e^c$. In this case, we have

$$\bar{R}^e \left( \frac{q_1}{q_e^c} - 1 \right) \leq \frac{q_1}{\bar{q}_e^R} - 1,$$

for any $q_1$, and the optimal value of $r_1$ is equal to $T_h$.

Finally, for $\bar{q}_e^c > \bar{q}_e^R$ and $\bar{q}_e^c < q_1$, (B274) is equivalent to

$$\frac{q_1}{\bar{q}_e^c} \left( \bar{R}^e - \frac{\bar{q}_e^c}{\bar{q}_e^R} \right) \leq \bar{R}^e - 1.$$

This conditions holds for any $q_1$ if $\bar{R}^e \leq \frac{\bar{q}_e^c}{\bar{q}_e^R}$. Note that $\bar{R}^e \leq \frac{\bar{q}_e^c}{\bar{q}_e^R}$ guarantees that $\Sigma_e \leq -1$.

On the other hand, if $\bar{R}^e > \frac{\bar{q}_e^c}{\bar{q}_e^R}$, then (B283) holds if and only if

$$q_1 \leq \bar{q}_e^c \left( \frac{1}{1 - \left( \frac{\delta R_e^r}{(1 - \delta) R_e^r T_h} \right) \left( \frac{\bar{q}_e^c}{\bar{q}_e^R} - 1 \right)} \right) = \frac{\bar{q}_e^c}{1 + \Sigma_e}.$$
Note that the right-hand side of (B284) is greater than or equal to $\bar{q}^e$ because $\bar{R}^e > \frac{d^e}{q^e}$ guarantees that $\Sigma_e > -1$. We can combine the result for both cases ($\bar{R}^e \leq \frac{d^e}{q^e}$ and $\bar{R}^e > \frac{d^e}{q^e}$) by using the notation $x^+ = \max(x,0)$ and requiring that

$$q_1 \leq \frac{\bar{q}^e}{(1 + \Sigma_e)^+},$$

as a condition for setting the optimal $r_1$ value to $T_h$.

We conclude by observing that (B285) is equivalent to (B275) when $\bar{q}^e \leq \bar{q}^R$.

c) Note that, as follows from the result in part b) of Proposition 3, if $\alpha^e > \bar{\alpha}^e(q_1)$, both patient groups become inflexible. In particular, we have $q_1 > q^+_e(c, \Delta, R^e, \alpha^e, \beta^e)$ and $q_2 > q^+_e(c, \Delta, R^e, \alpha^e, \beta^e)$. Then, both patient groups insist on the shortest possible revisit interval, and $\hat{r}^e_1 = \hat{r}^e_2 = T_l$.

\[ \square \]

**Proof of Corollary B2**

In the “proportional” setting we have $\bar{q}^R_i = \bar{q}^e$, so that

$$\frac{R^e_i}{A^e_i} = 1, i = 1, 2.$$

Then, $\bar{R}^e = 1 + \frac{(1-\delta)\bar{R}^e R^e_t}{\delta R^e_t} > 1 = \frac{R^e_t}{A^e_t}, i = 1, 2$.

Then, if $\alpha^e \leq \bar{\alpha}^e(q_2)$, then, as (B19) and (B20) imply, $\hat{r}^e_1 = T_h$ whenever $q_1 \leq \bar{q}^e$, and $\hat{r}^e_1 = T_l$ otherwise, while $\hat{r}^e_2 = T_h$ whenever $q_2 \leq \bar{q}^e$, and $\hat{r}^e_2 = T_l$ otherwise. Also, if $\alpha^e(q_1) > \bar{\alpha}^e(q_2)$ holds, then $\hat{r}^e_1 = T_l$ and, as (B21) implies, $\hat{r}^e_1 = T_h$ if and only if $q_1 \leq \bar{q}^e$. Finally, if $\alpha^e > \bar{\alpha}^e(q_1)$, both $\hat{r}^e_1$ and $\hat{r}^e_2$ are set at $T_l$.

\[ \square \]
Appendix C: Summary of Modeling Notation

The Table below summarizes the notation we use in the model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N, N^e$</td>
<td>Physician panel size w/o and w/ e-visits, respectively</td>
</tr>
<tr>
<td>$r, r^e$</td>
<td>Patient revisit interval (RVI) w/o and w/ e-visits, respectively</td>
</tr>
<tr>
<td>$T$</td>
<td>Time until the patient falls sick following an office visit in the absence of care</td>
</tr>
<tr>
<td>$T_l, T_h$</td>
<td>The finite support of $T$</td>
</tr>
<tr>
<td>$q$</td>
<td>The probability that a patient falls sick $T_l$ time after an office visit in the absence of care</td>
</tr>
<tr>
<td>$c_o$</td>
<td>Cost of a “routine” office visit for patient</td>
</tr>
<tr>
<td>$\eta$</td>
<td>The additional cost associated with the patient being sick when visiting the office</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Patient flexibility parameter</td>
</tr>
<tr>
<td>$c$</td>
<td>Center of the interval for the set of $\eta$ values</td>
</tr>
<tr>
<td>$\rho(r)$</td>
<td>Probability that a particular office visit falls into the routine category</td>
</tr>
<tr>
<td>$\Pi_\delta(N, r), \Pi_\delta^e(N, r)$</td>
<td>Physician total compensation w/o and w/ e-visits, respectively</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The proportion of physician daily compensation that is based on the FFS scheme</td>
</tr>
<tr>
<td>$A$</td>
<td>Physician’s total daily capacity</td>
</tr>
<tr>
<td>$\tau^r, \tau^u, \tau^e$</td>
<td>Physician time required for routine, urgent, and e-visit appointments, respectively</td>
</tr>
<tr>
<td>$R^r, R^u, R^e$</td>
<td>Physician compensation for each routine, urgent, and e-visit appointment, respectively</td>
</tr>
<tr>
<td>$R^d, R^d_e$</td>
<td>Physician daily compensation for each patient on the panel for office and e-visits, respectively</td>
</tr>
<tr>
<td>$\alpha_e$</td>
<td>The average fraction of routine visits that the patient and physician attempt to replace with e-visits</td>
</tr>
<tr>
<td>$\beta_e^r$</td>
<td>The average fraction of e-visit attempts that are successful</td>
</tr>
<tr>
<td>$\bar{\theta}$</td>
<td>Patient’s decision variable to sign up for e-visits</td>
</tr>
</tbody>
</table>

Table C1: List of Notation